

# **Mechanisms Of The Distributed Practice Effect**

**Michael Mozer**

**University of Colorado**

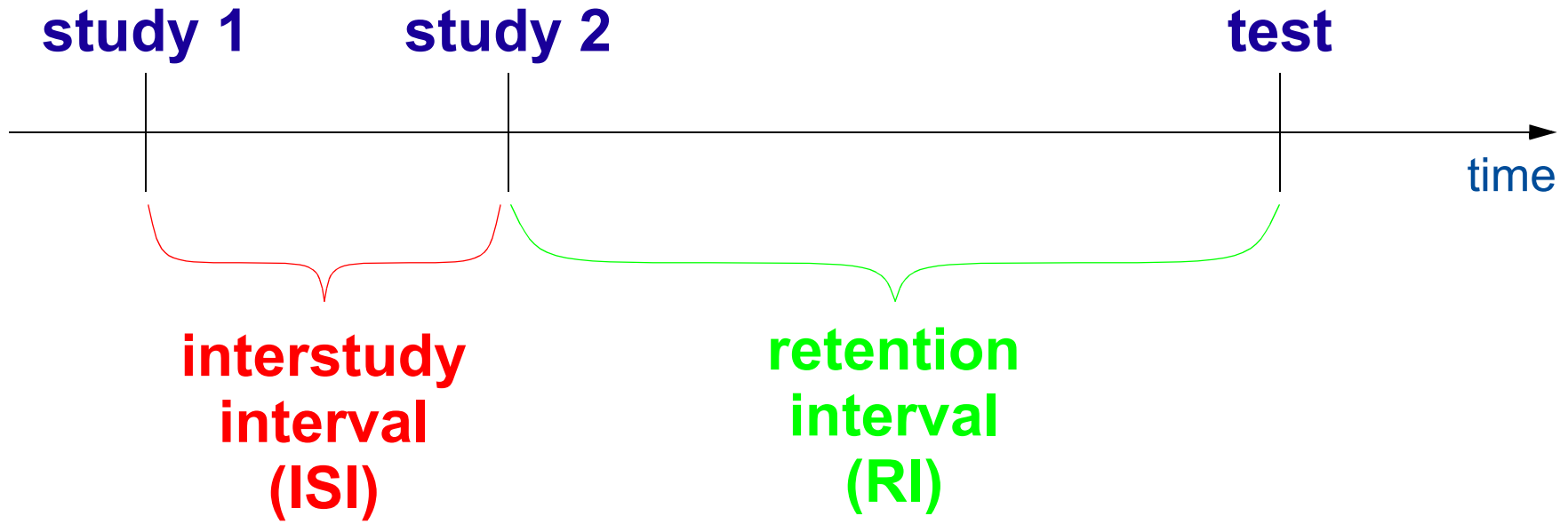
**Harold Pashler**

**UCSD**

# Reminder: The Basic Paradigm



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# Rich Theoretical Literature Attempts to Explain Distributed Practice Effect

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**Raaijmakers (2003)**

- **Predictive utility**

**Staddon, Chelaru, & Higa (2002)**

# Rich Theoretical Literature Attempts to Explain Distributed Practice Effect

- **Encoding variability**
- **Predictive utility**

**Raaijmakers (2003)**

**+**

**Staddon, Chelaru, & Higa (2002)**

**=**

**cool story about the  
temporal dynamics of memory**

# Encoding Variability Theories

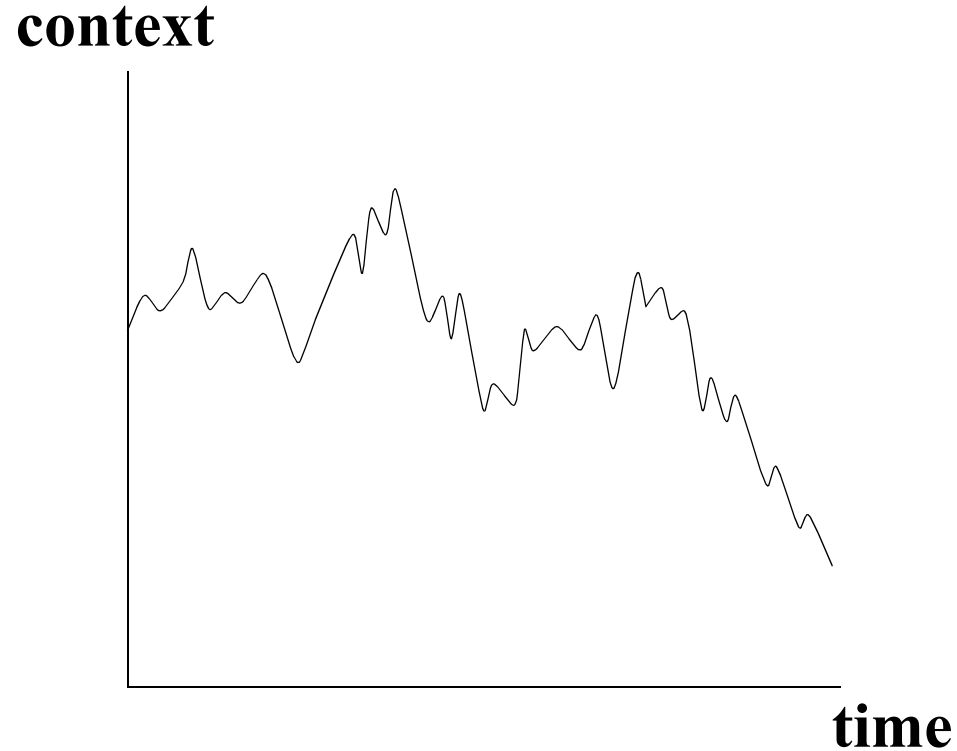


# Encoding Variability Theories

Each study episode, a separate trace is laid down.

The trace includes a psychological *context*.

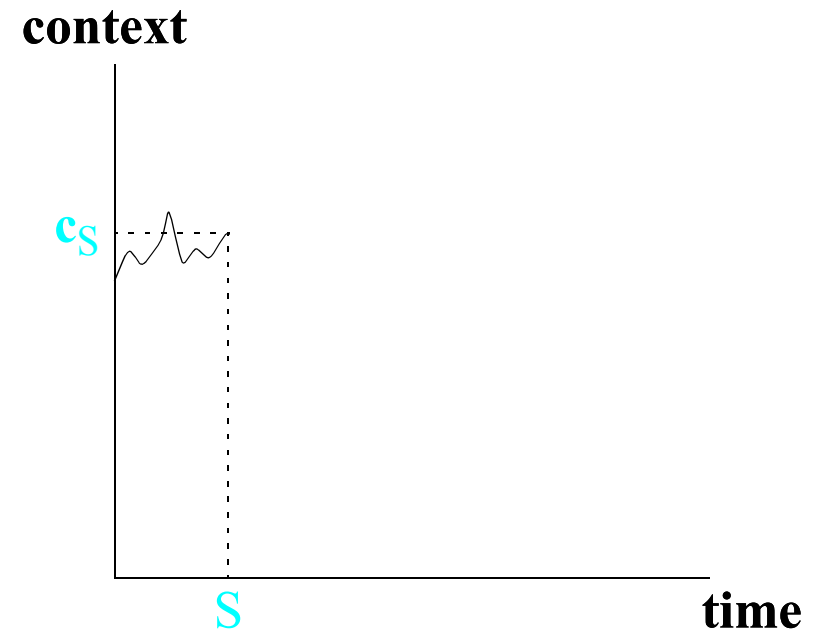
Context wanders over time.





# Encoding Variability Explains Forgetting

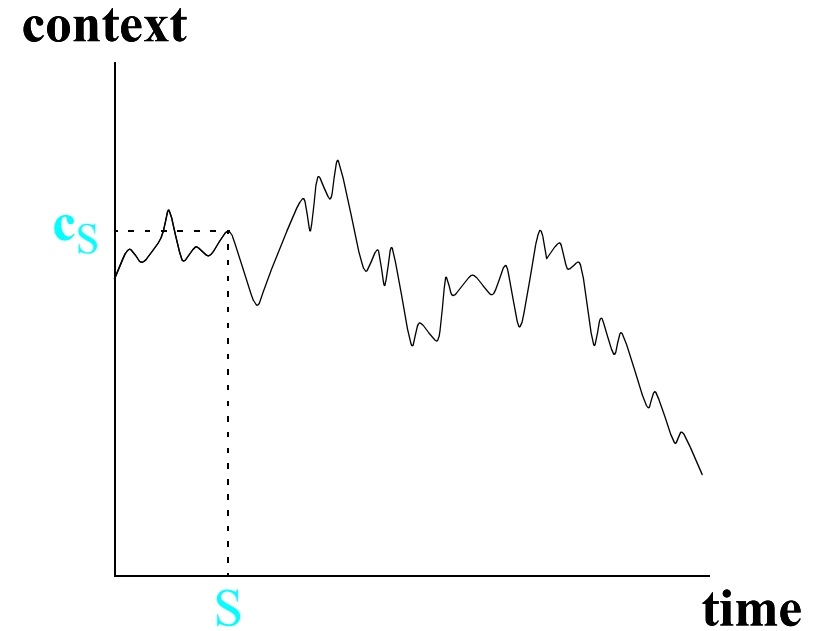
Study item at **S**



# Encoding Variability Explains Forgetting

Study item at **S**

During retention interval, context wanders



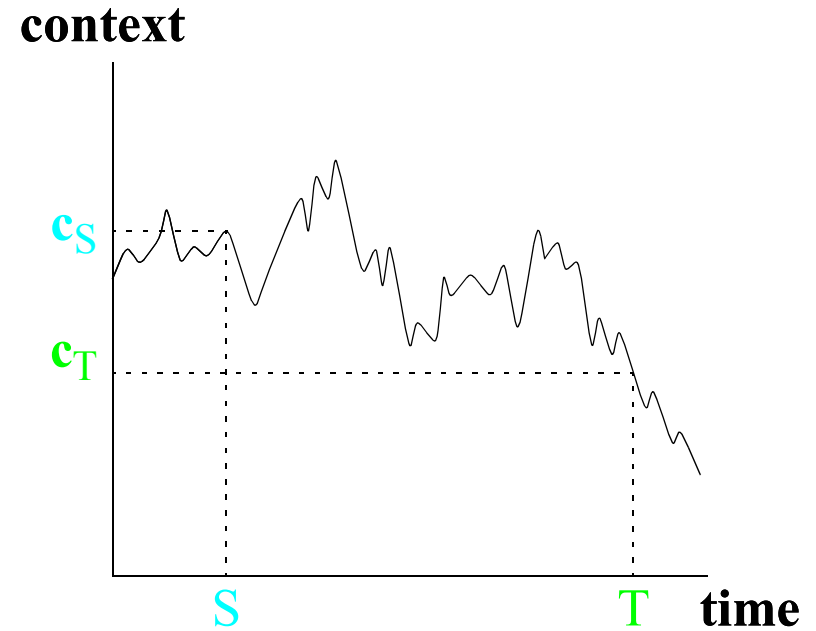
# Encoding Variability Explains Forgetting

Study item at **S**

During retention interval, context wanders

Test at **T**

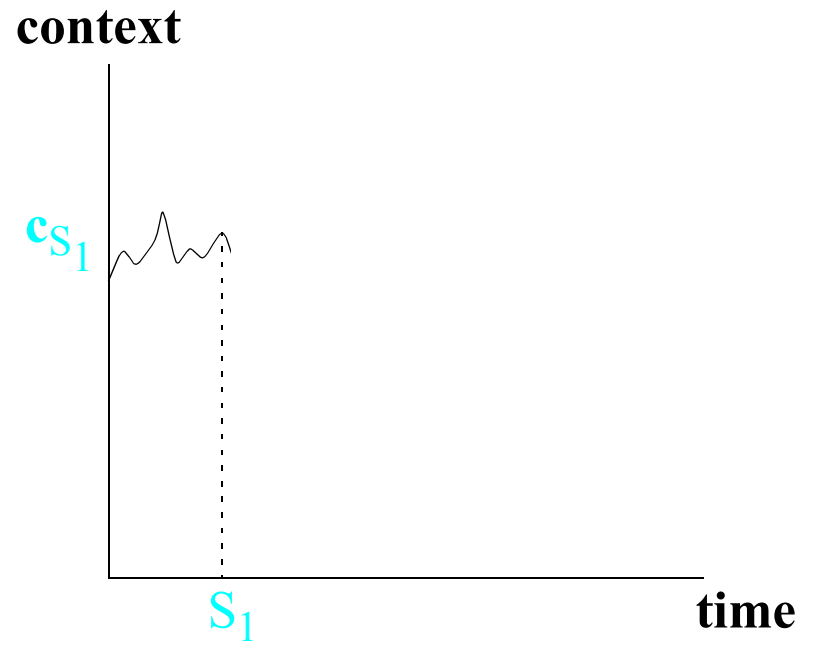
Retrieval success depends on similarity of **c<sub>T</sub>** and **c<sub>S</sub>**



# Encoding Variability Explains DP Effect

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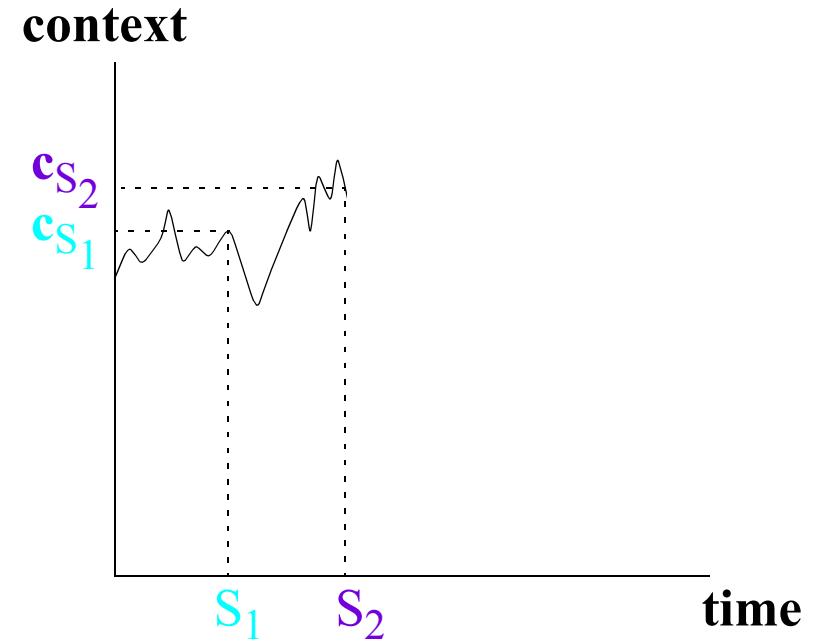
Study item at **S1**



# Encoding Variability Explains DP Effect

Study item at **S1**

Study item at **S2**





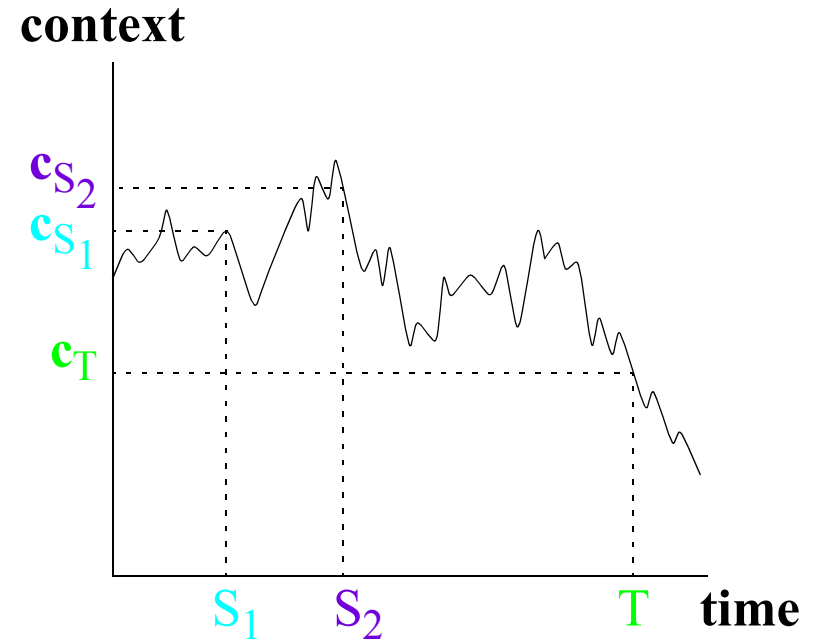
# Encoding Variability Explains DP Effect

Study item at **S1**

Study item at **S2**

Test at **T**

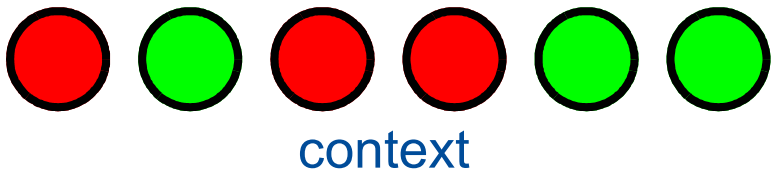
Retrieval success at **T** depends on similarity of **c<sub>T</sub>** to either **c<sub>S1</sub>** or **c<sub>S2</sub>**



Disadvantage for small ISIs: redundancy of **c<sub>S1</sub>** and **c<sub>S2</sub>**.

# Raaijmackers (2003)

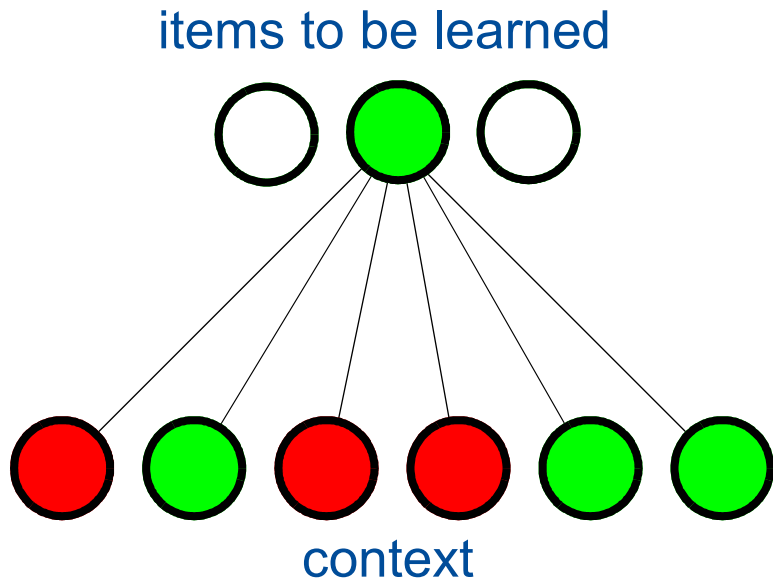
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Each item to be learned represented by an output neuron.

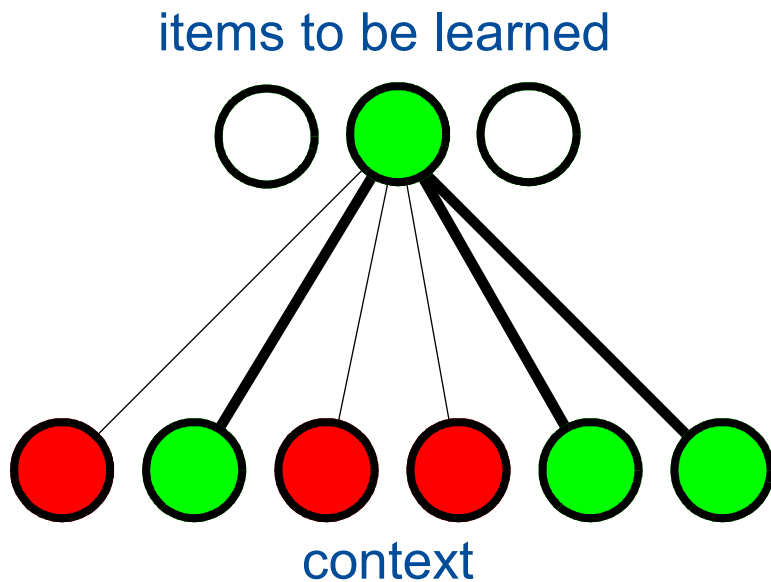


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## Hebbian learning rule



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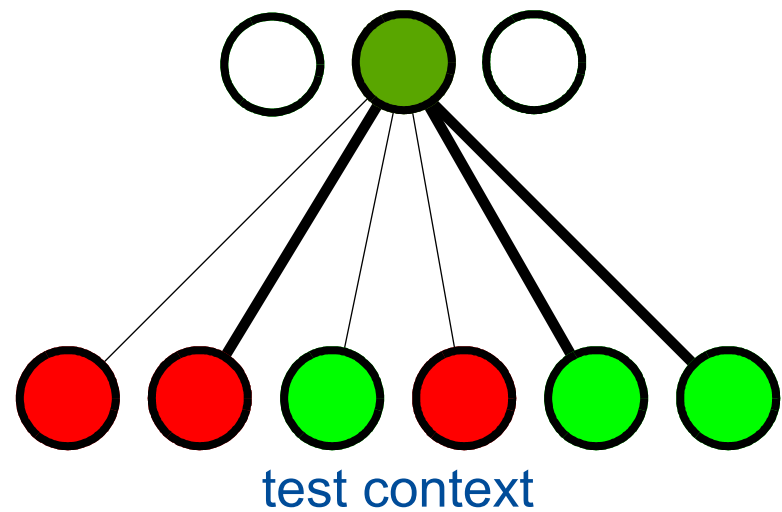
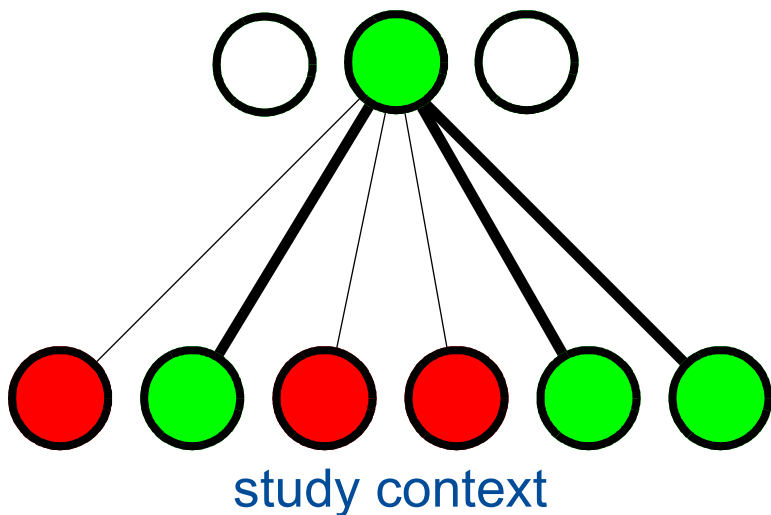
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Hebbian learning rule

**Output activity at test  $\sim$  recall probability**

depends on similarity of study and test contexts



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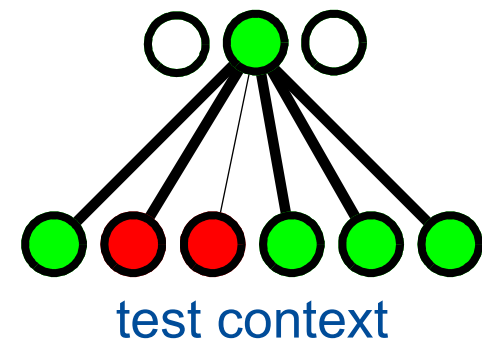
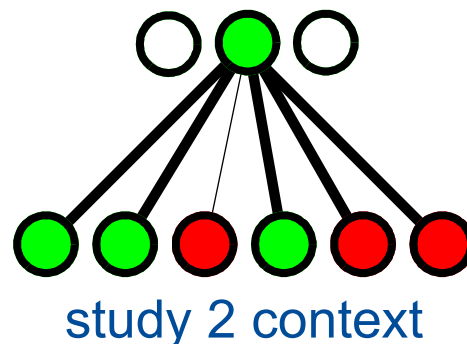
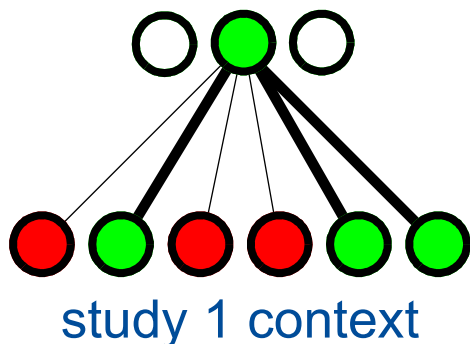
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Hebbian learning rule

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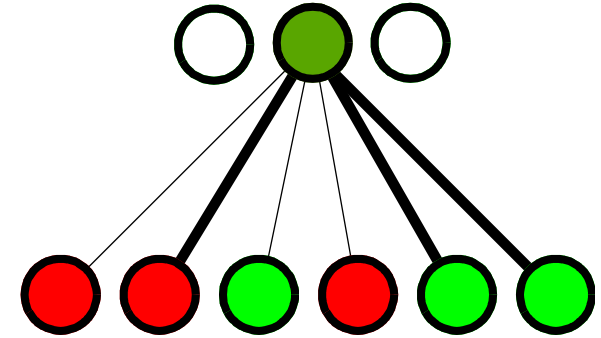
**Multiple study opportunities  $\Rightarrow$  context variability  
 $\Rightarrow$  robust recall**



# Raaijmakers (2003): Formal Description

**Retrieval at test facilitated when context unit active at both study and test.**

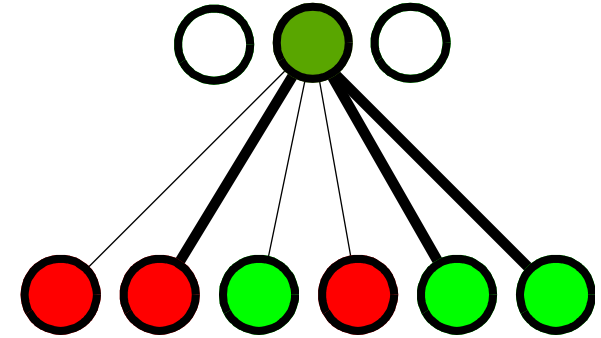
Expected output neuron activity ~  
 $P(\text{retrieval}) \sim P(C_S = 1 \ \& \ C_T = 1)$



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## How does context wander over time?

context bits flip from off to on at rate  $\mu_{01}$

context bits flip from on to off at rate  $\mu_{10}$

$$P(C_S = 1 \ \& \ C_T = 1) = \beta^2 + \beta(1-\beta) \exp(-\alpha \text{RI})$$

retention interval

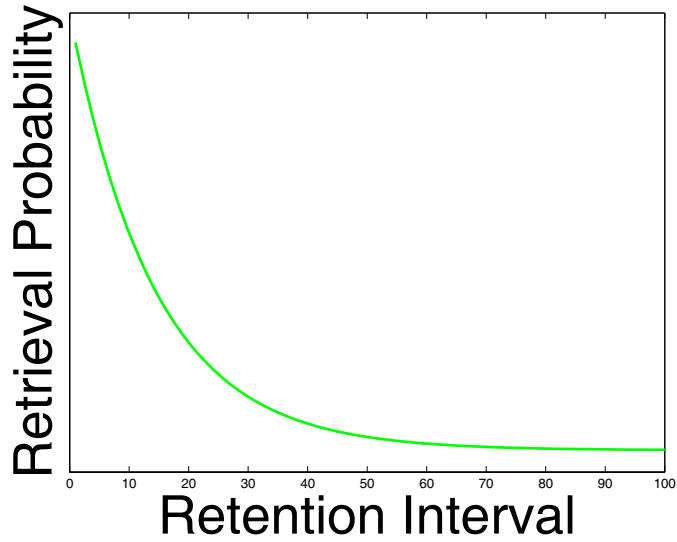
flip rate:  $\mu_{01} + \mu_{10}$

proportion on:  $\mu_{01} / (\mu_{01} + \mu_{10})$



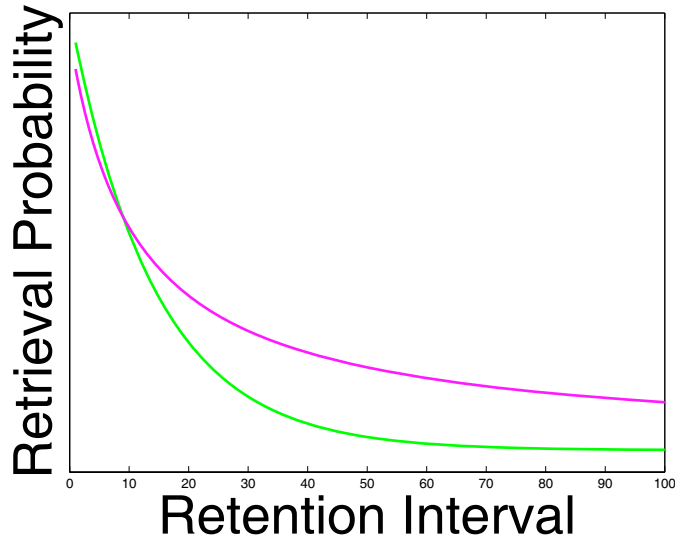
# What It Boils Down To

Forgetting function is **exponential**



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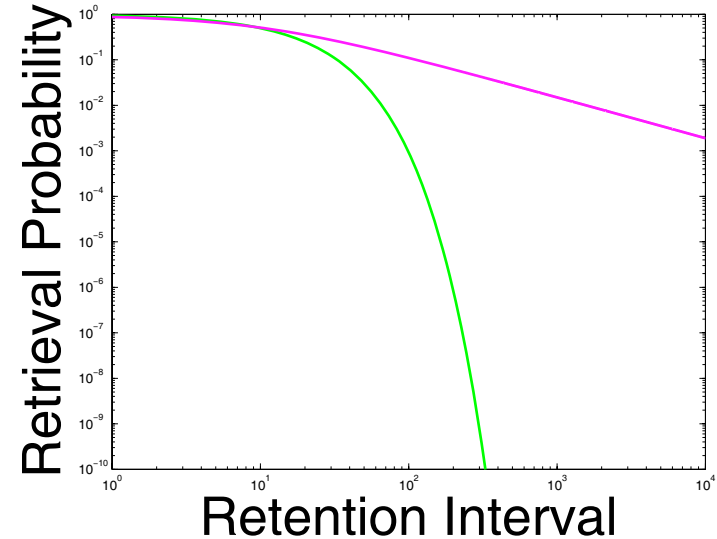
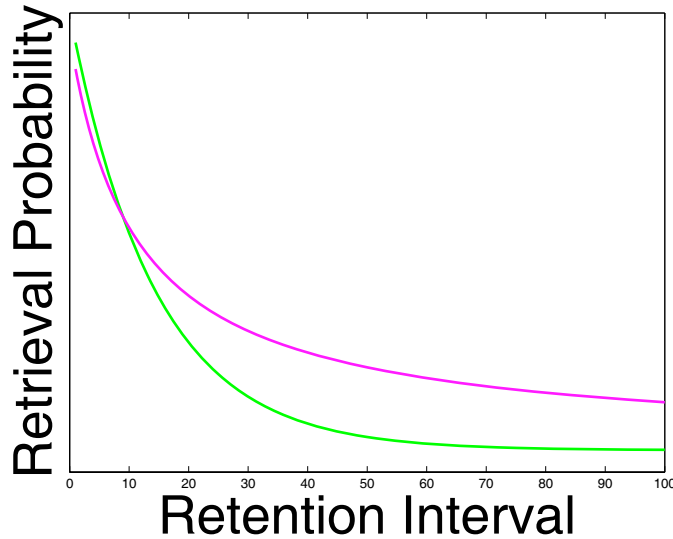


Human forgetting functions follow a **power law** (Wickelgren, 1974; Wixted & Carpenter, 2007):

$$P(\text{retrieval}) = \lambda(1 + \varphi \text{RI})^{-\phi}$$

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**Power law shows scale invariance**

I.e., memory shows same properties at different time scales

# Is it a problem that Raaijmakers' (2003) model doesn't show scale invariance?

Yes, distributed practice effects are scale invariant.

## Model has other problems too.

- Many free parameters and ugly hacks
- Doesn't fit data particularly well

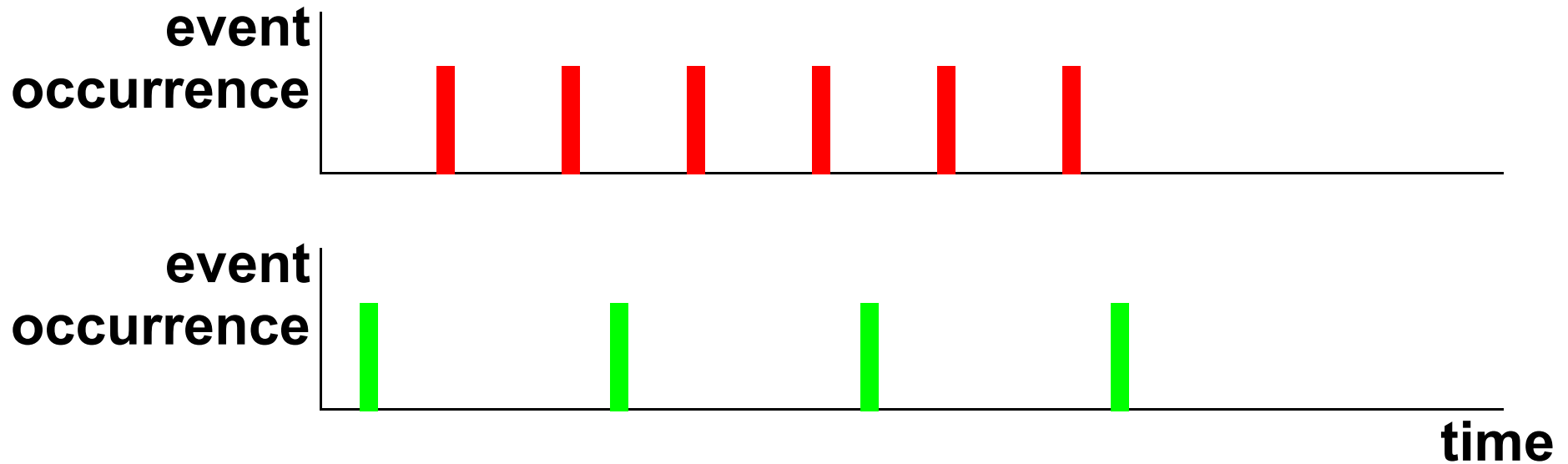
# Predictive Utility Theories

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Suppose that memory

- is limited in capacity, and/or
- is imperfect and allows intrusions.

To achieve optimal performance, memories should be erased if they are not likely to be needed in the future.

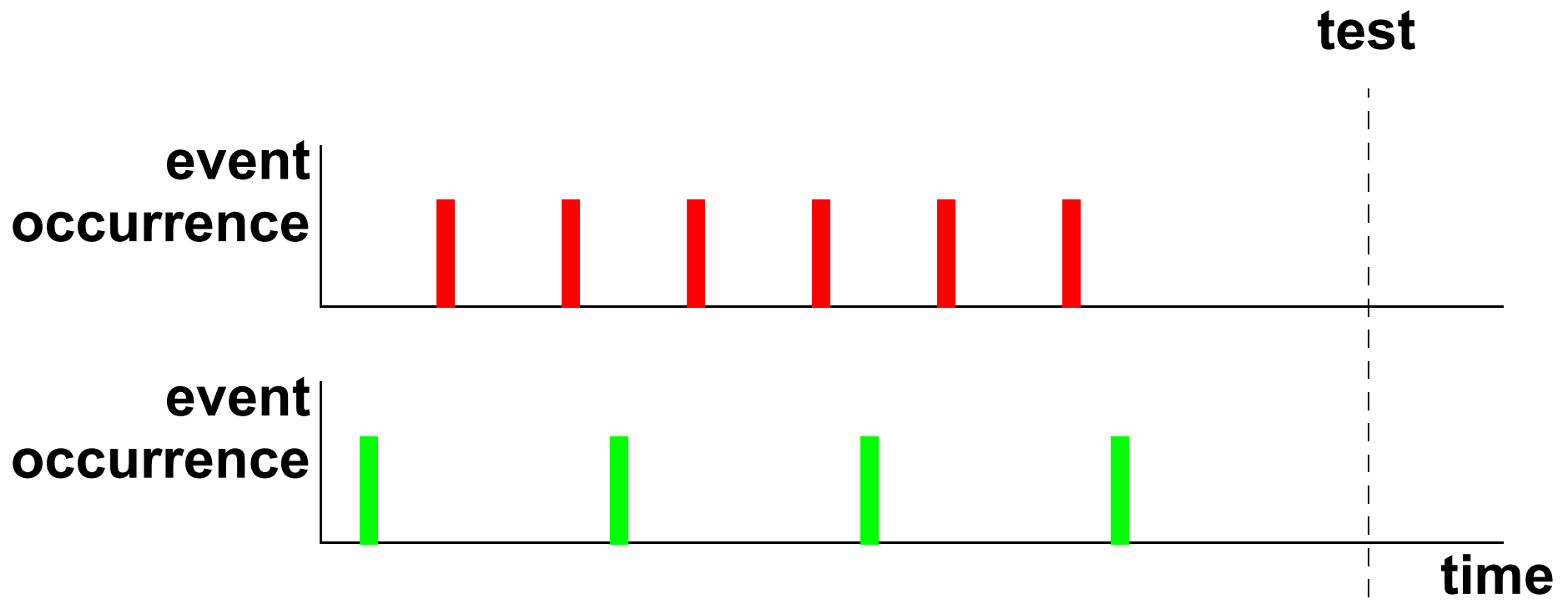


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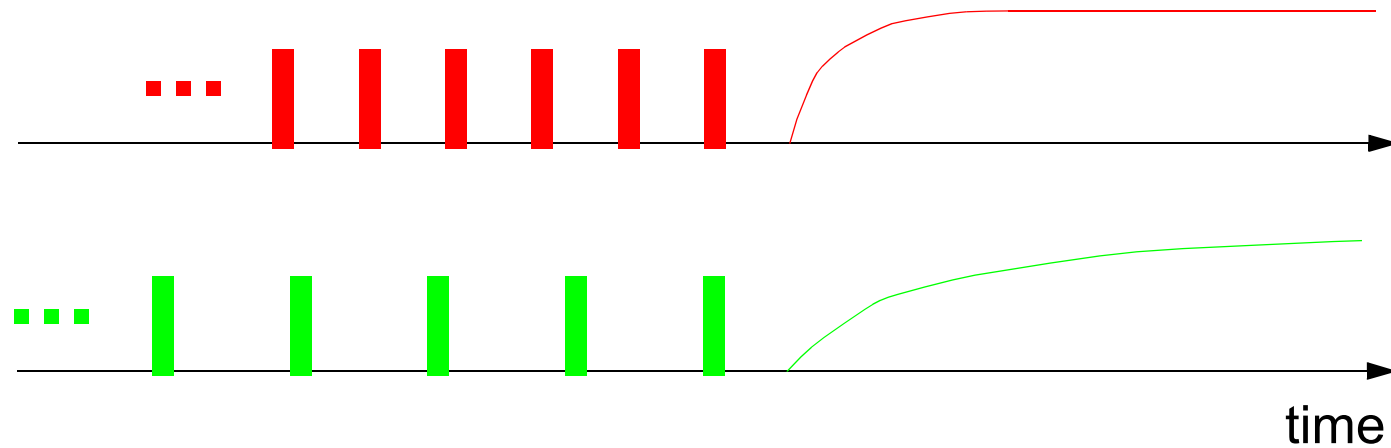


# Staddon, Chelaru, & Higa (2002)

Rats habituate to a repeated stream of stimuli.

Time for recovery from habituation  $\sim$  rate of stimuli

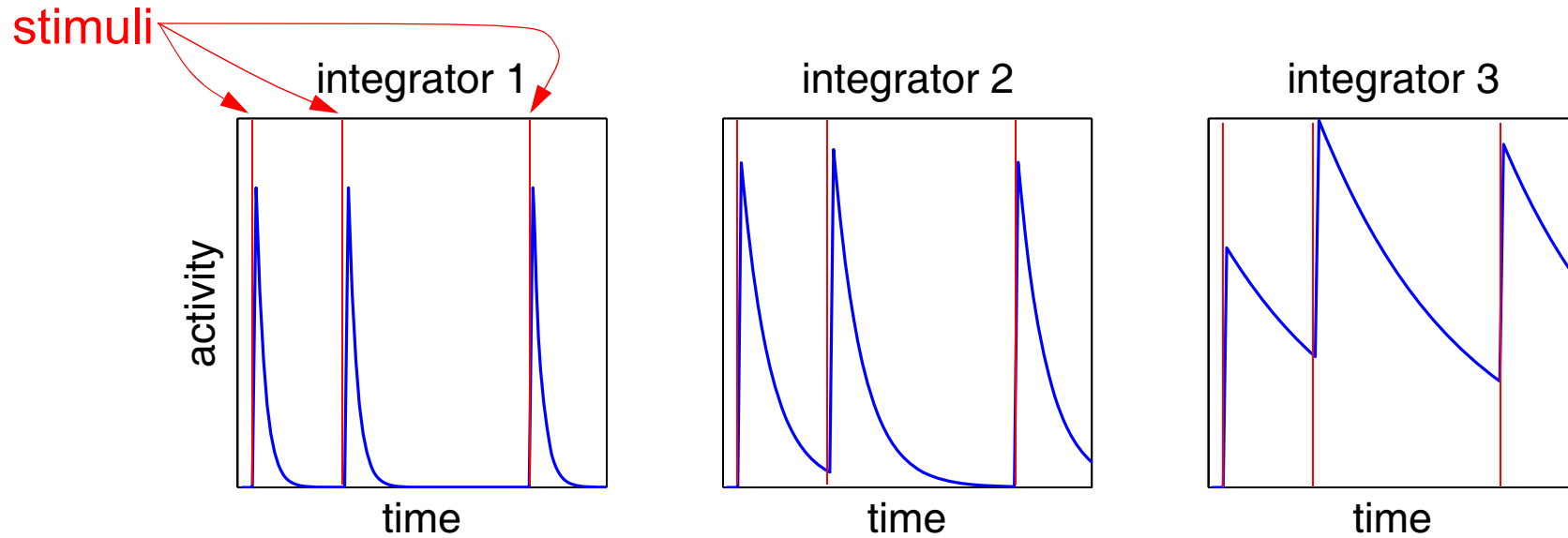
Longer-lasting memory for stimuli delivered at slower rate





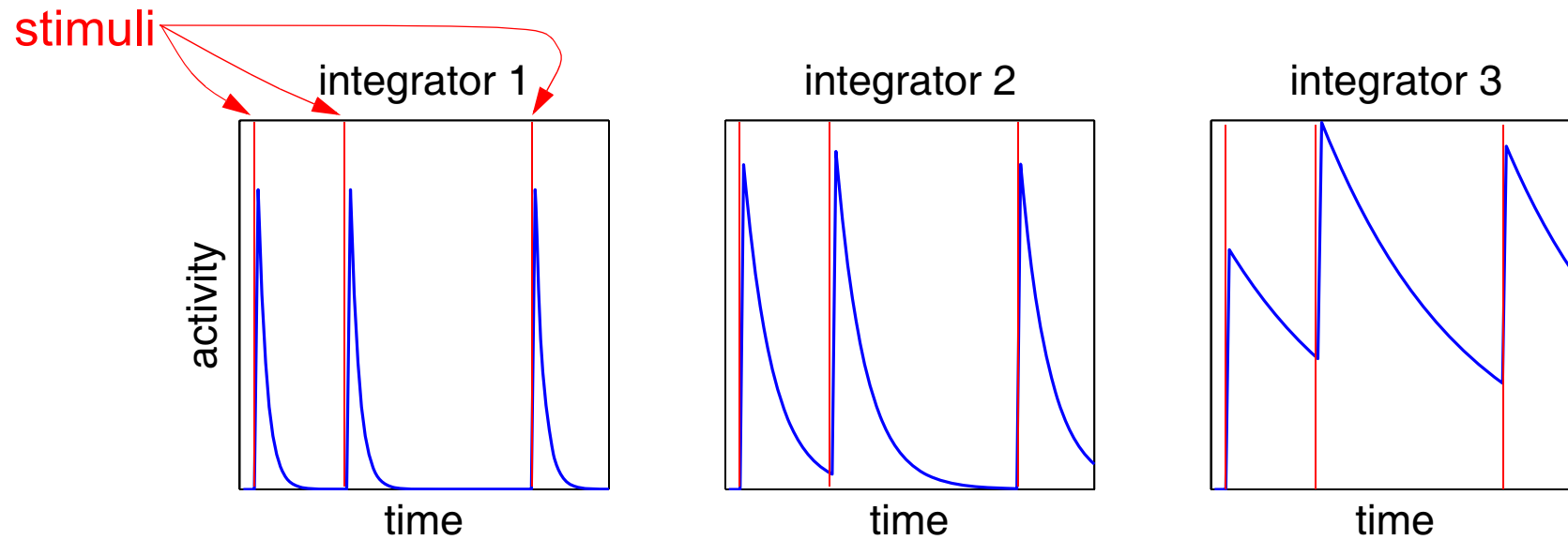
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Each item to be learned represented by memory consisting of *leaky integrators* at multiple time scales.



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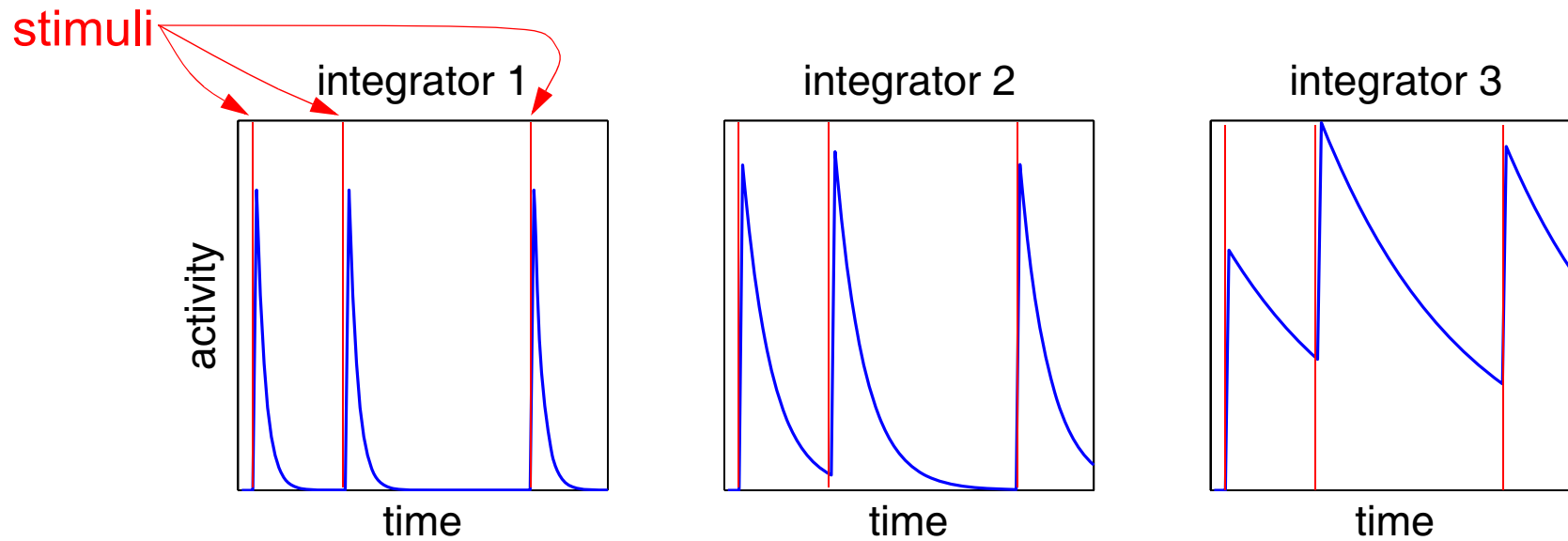
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**Memory trace is the sum of the integrator activities.**

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Memory trace is the sum of the integrator activities.

## Memory storage rule

Integrators with long time constants get activated only when integrators with short time constants have decayed.

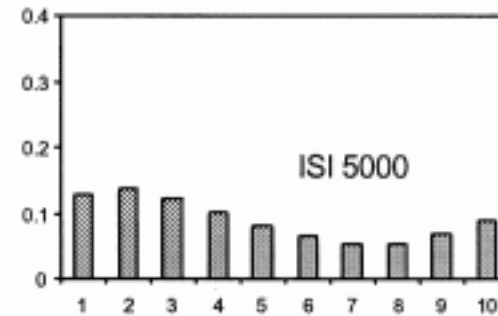
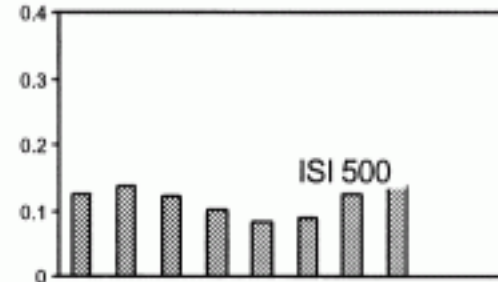
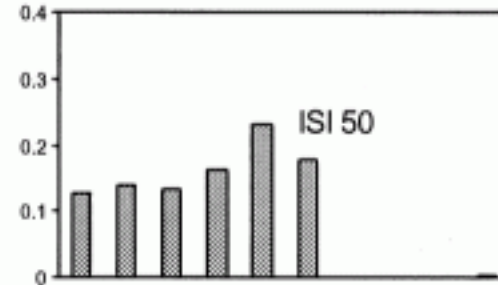
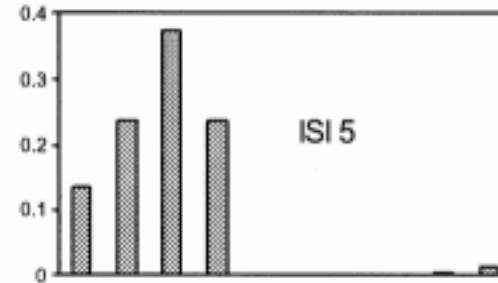
# Example

10 integrators

Stimulus repeatedly  
presented at various ISIs

Greater spacing  $\Rightarrow$   
memory shifts to longer  
time-scale integrators  $\Rightarrow$   
more durable memory

Activation (arbitrary units)



# Example

10 integrators

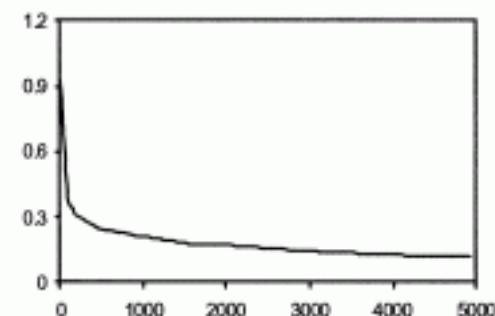
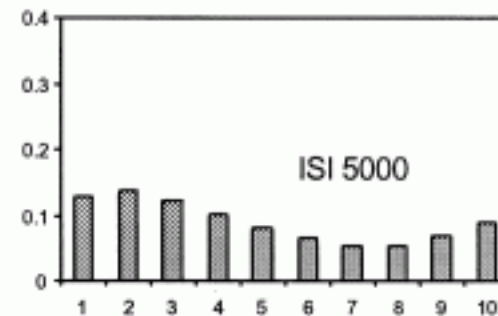
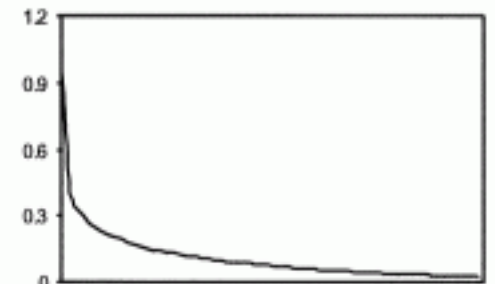
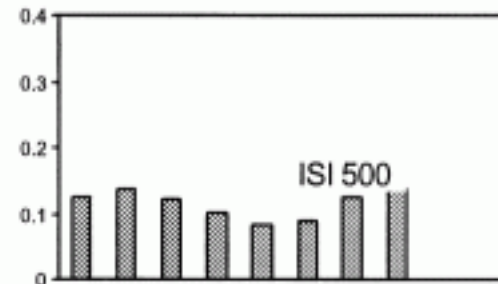
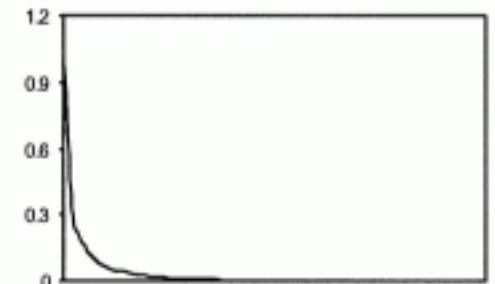
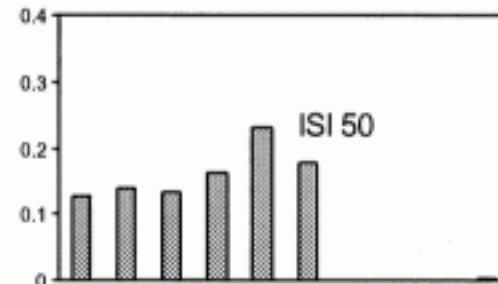
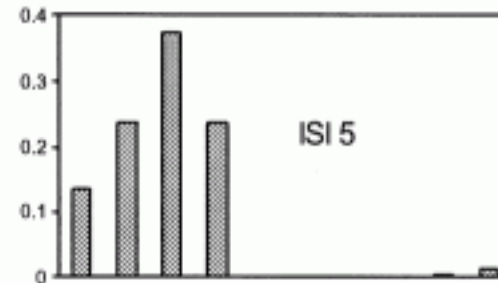
Stimulus repeatedly  
presented at various ISIs

Greater spacing  $\Rightarrow$   
memory shifts to longer  
time-scale integrators  $\Rightarrow$   
more durable memory

**Model is sensitive to  
predictive utility**

Slower forgetting following  
longer ISI stimulus sequences.

Activation (arbitrary units)

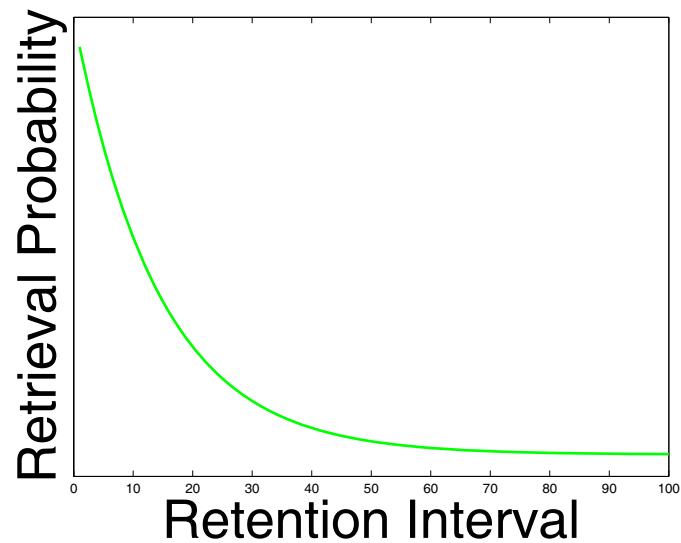


**Model was fit to rat habituation and interval timing data,**

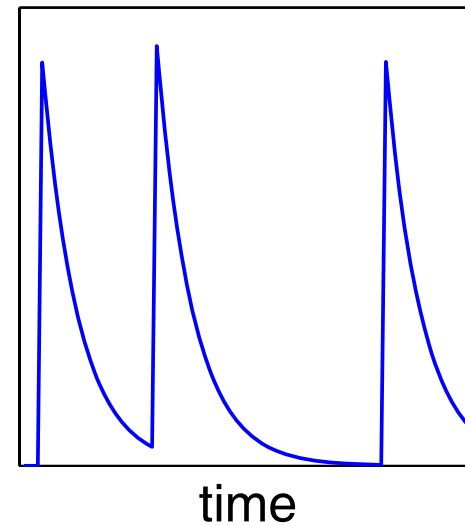
**... but isn't sufficiently well specified to explain human studies of distributed practice.**

# Two Models Share Key Property: Exponential Decay of Internal Representations

encoding variability model  
via context drift



predictive utility model  
via leaky integrator



**This commonality allows us to integrate the two models.**

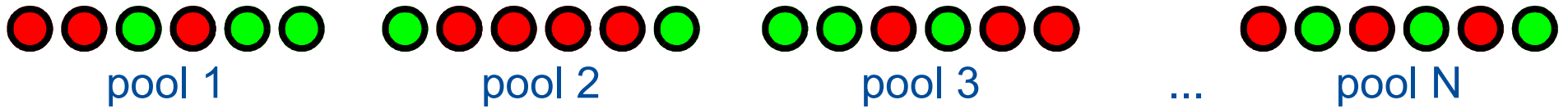
## **Combine**

- **multiple time-scale representation of Staddon's model**
- **contextual drift of Raaijmakers' model**

→ **Multiscale Context Model**



# Multiscale Context Model (MCM)

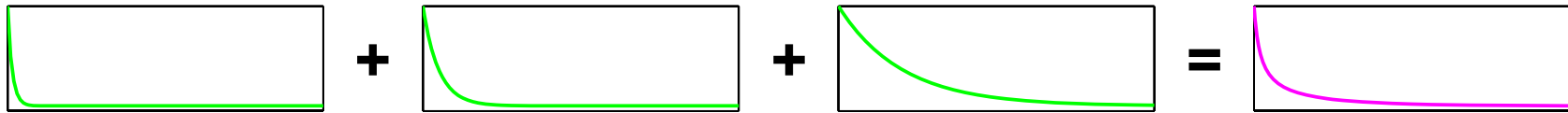


In pool  $p$ , all units flip state at rate  $\alpha_p$ .

The pools can be different sizes:  
the relative proportion of units in pool  $p$  is  $\gamma_p$ .

Retrieval function is a mixture of exponentials.

$$P(\text{retrieval}) \sim \sum_p \gamma_p \exp(-\alpha_p RI)$$

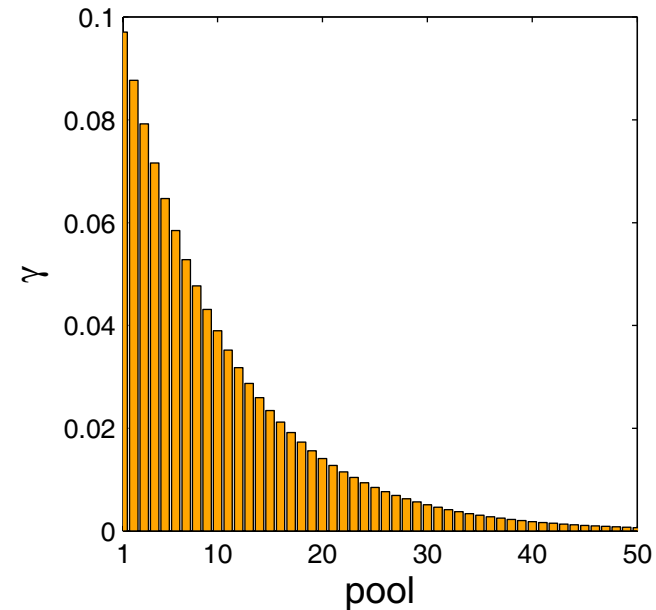
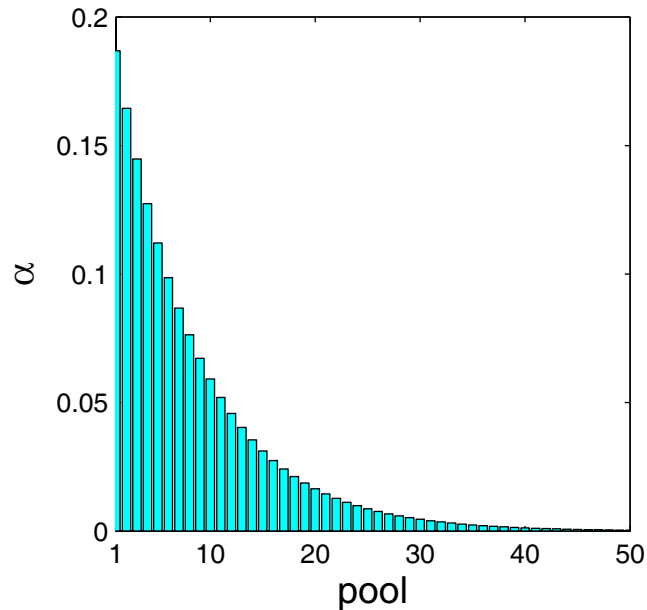


Mixture of exponentials can approximate human forgetting functions (Wixted).

# Use Simple Formula to Pick Pool Size ( $\gamma$ ) and Rate ( $\alpha$ )

$$\alpha_p = \mu v^p \quad \text{for } p \in [1, N]$$

$$\gamma_p = \omega^p$$

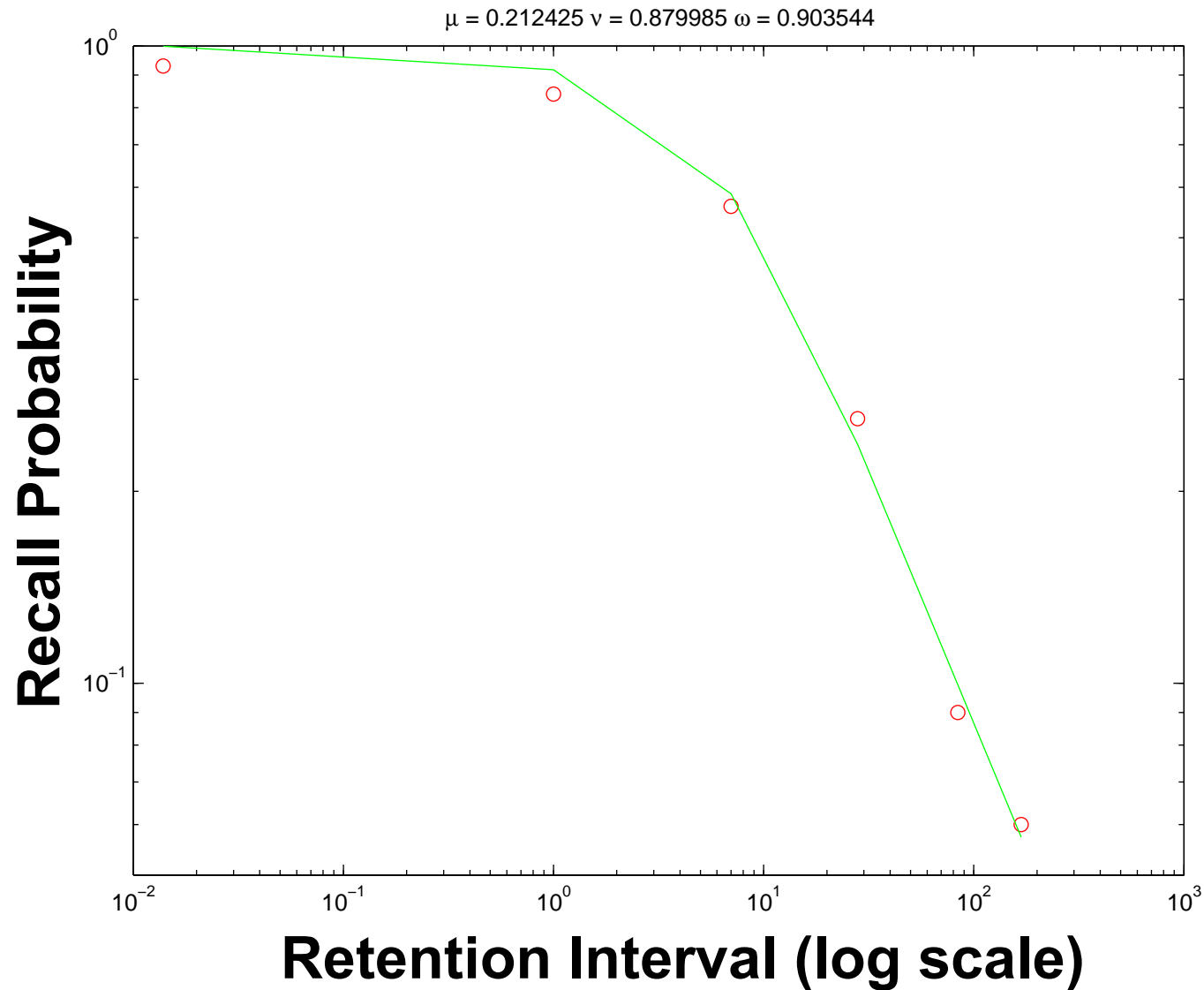


**MCM has four free parameters ( $\mu$ ,  $v$ ,  $\omega$ , + one more)**

**Can we select these parameters such that resulting model yields power law forgetting function and good fits to human data?**

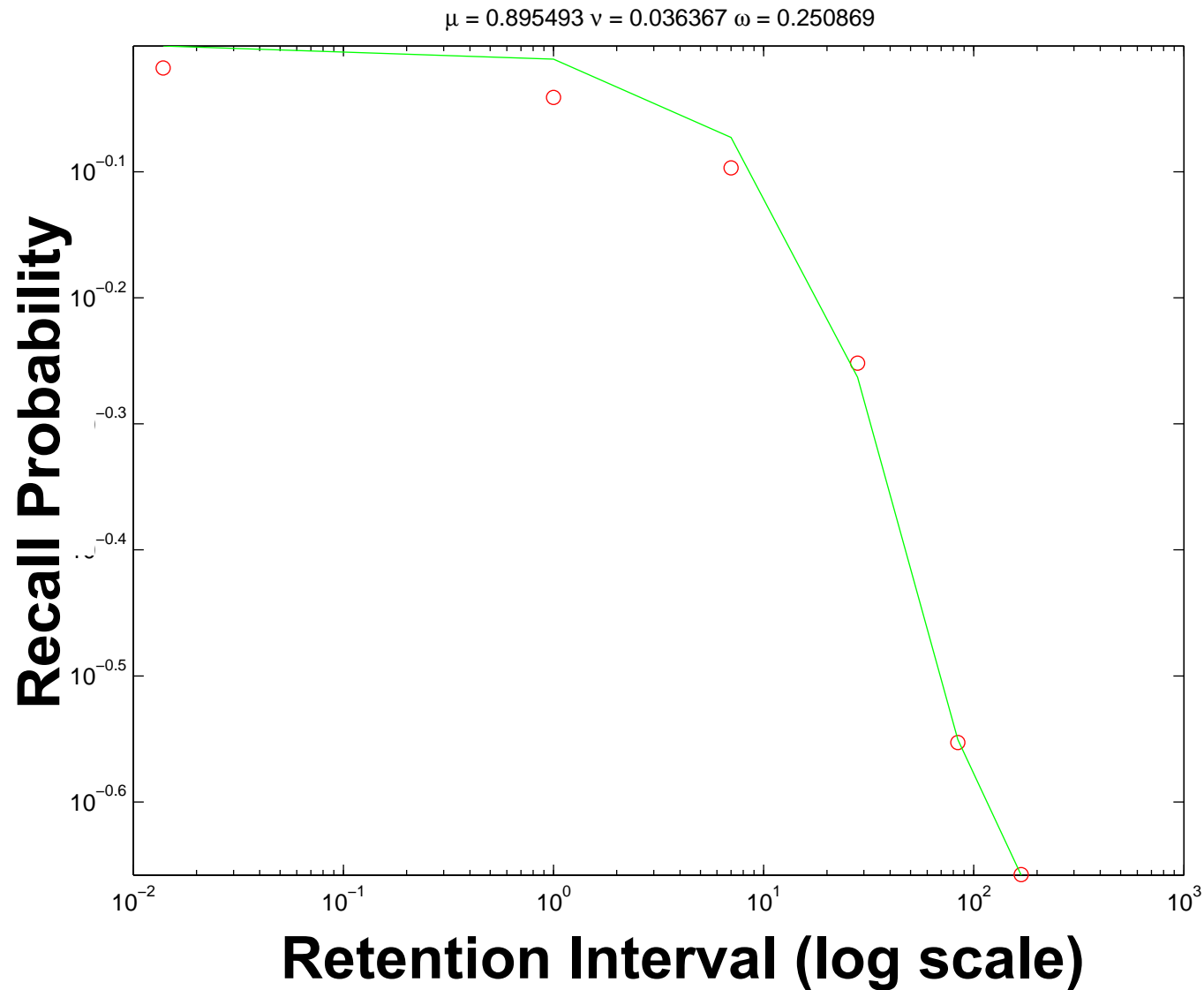
# Fitting Forgetting Functions I

Cepeda et al., Expt 2B



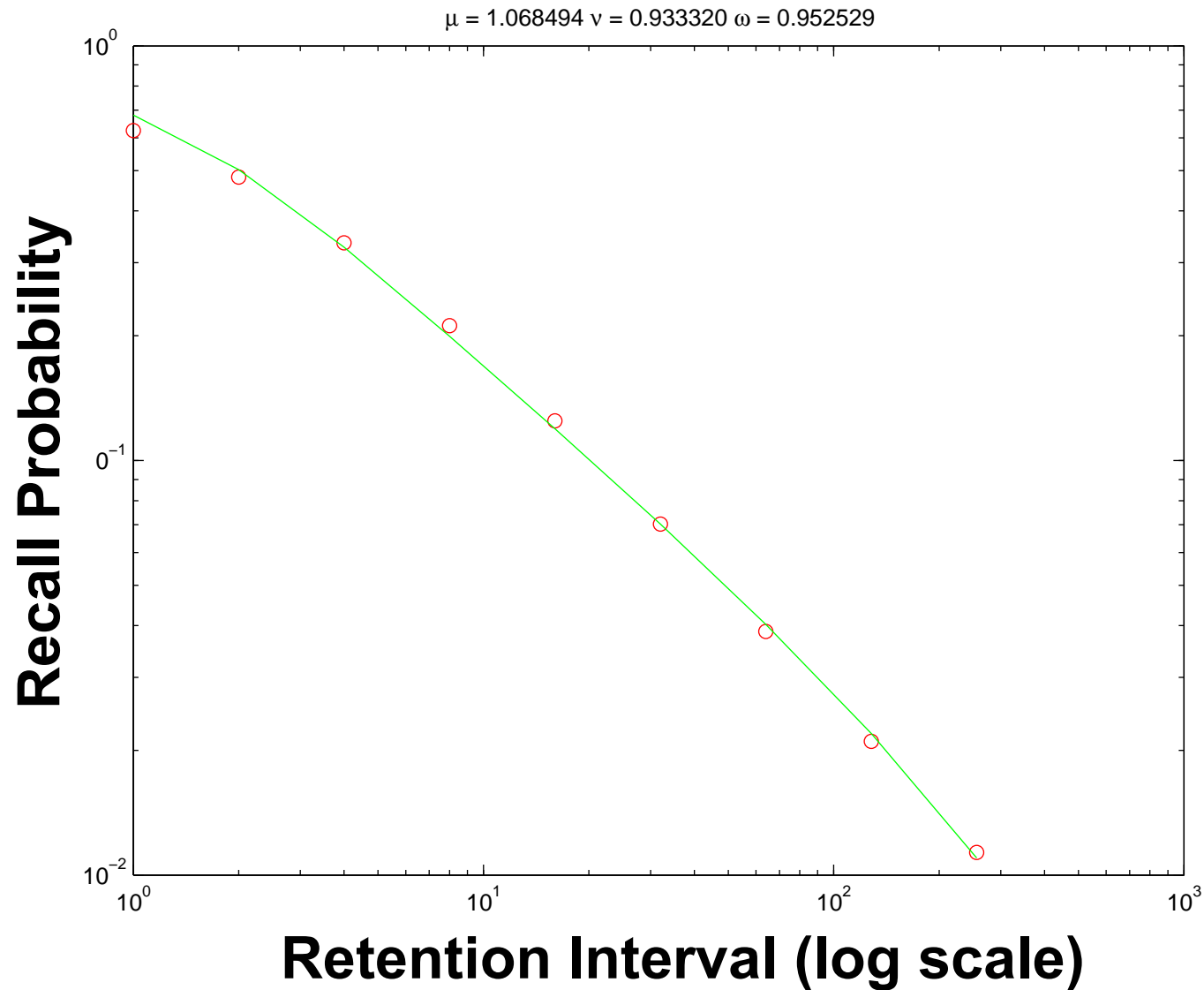
# Fitting Forgetting Functions II

## Cepeda et al., Expt 2A



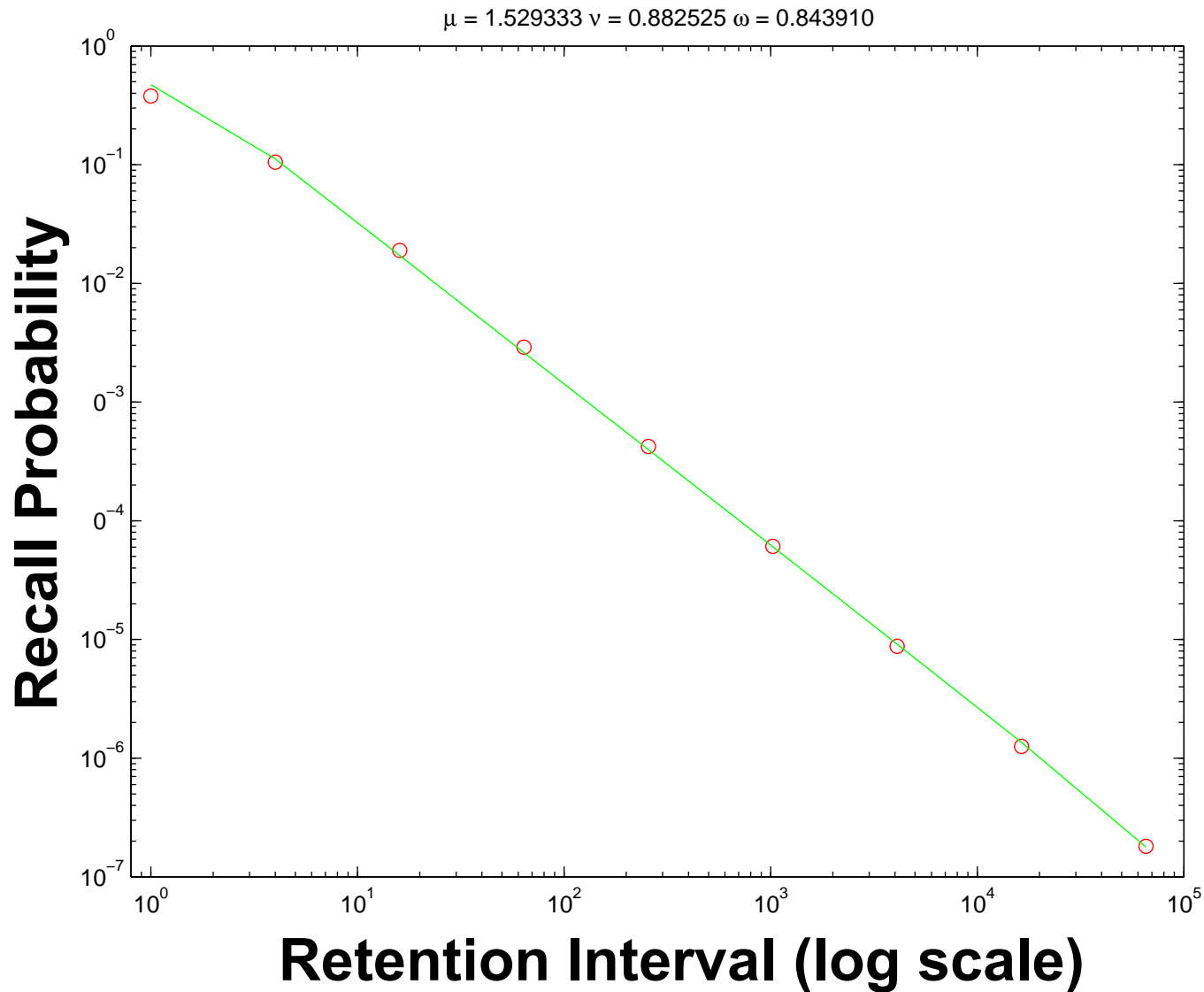
# Fitting Forgetting Functions III

$$P(\text{recall}) = .9(1 + .5 t)^{-0.9}$$



# Fitting Forgetting Functions IV

$$P(\text{recall}) = (1 + t)^{-1.4}$$



# Multiscale Context Model: A Convergence of Theories

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	<b>Raaijmakers (2003)</b>	<b>Staddon et al. (2002)</b>
<b>context drift</b>	<b>X</b>	
<b>multiple time-scale representation</b>		<b>X</b>
<b>learning rule</b>	<b>X</b> (dependence of learning on retrieval success)	<b>X</b> (cascaded error correction)

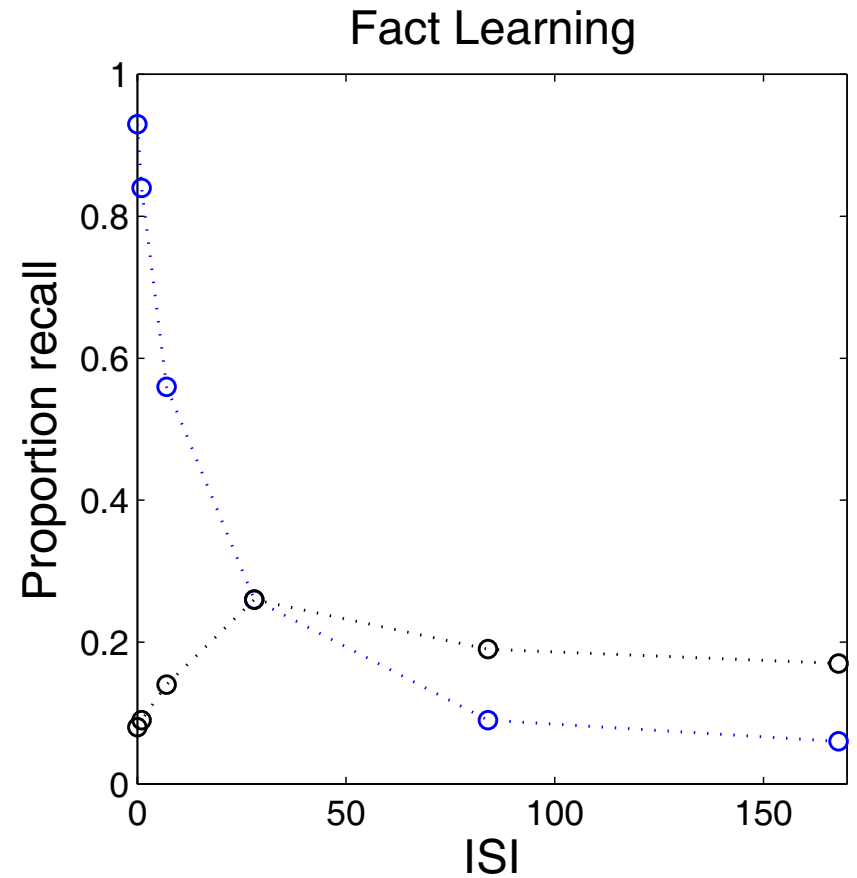
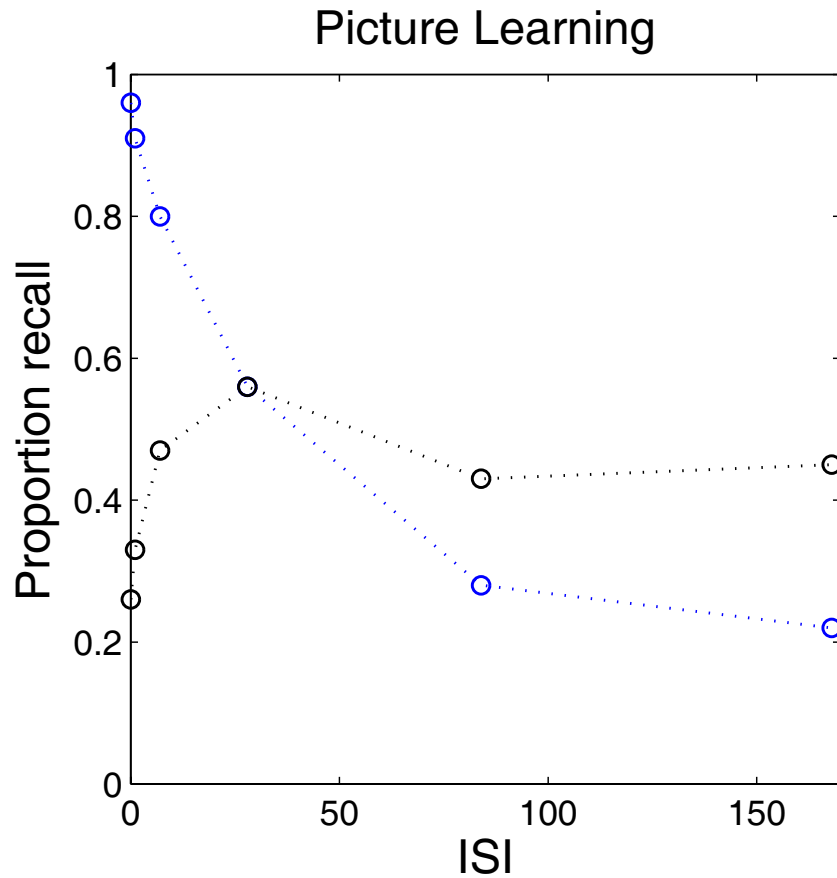


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	<b>Raaijmakers (2003)</b>	<b>Staddon et al. (2002)</b>	<b>Our Contribution</b>
<b>context drift</b>	<b>X</b>		
<b>multiple time-scale representation</b>		<b>X</b>	
<b>learning rule</b>	<b>X</b> (dependence of learning on retrieval success)	<b>X</b> (cascaded error correction)	
<b>variable pool size</b>			<b>X</b>
<b>parameterization of multiscale constants</b>			<b>X</b>
<b>neural characterization</b>			<b>X</b>

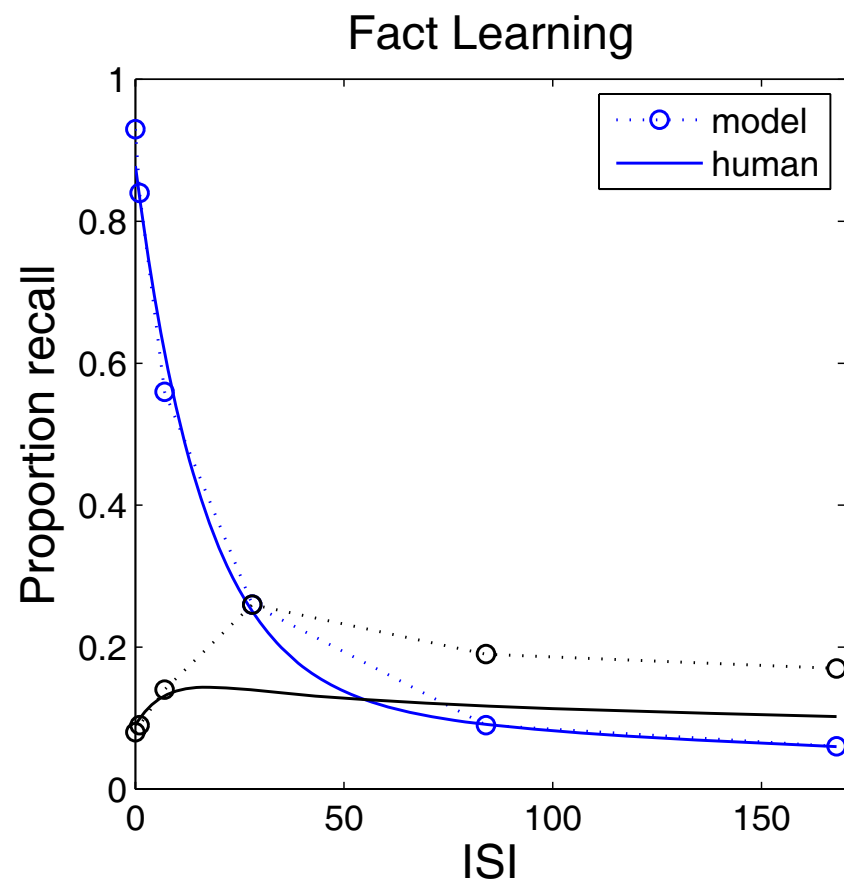
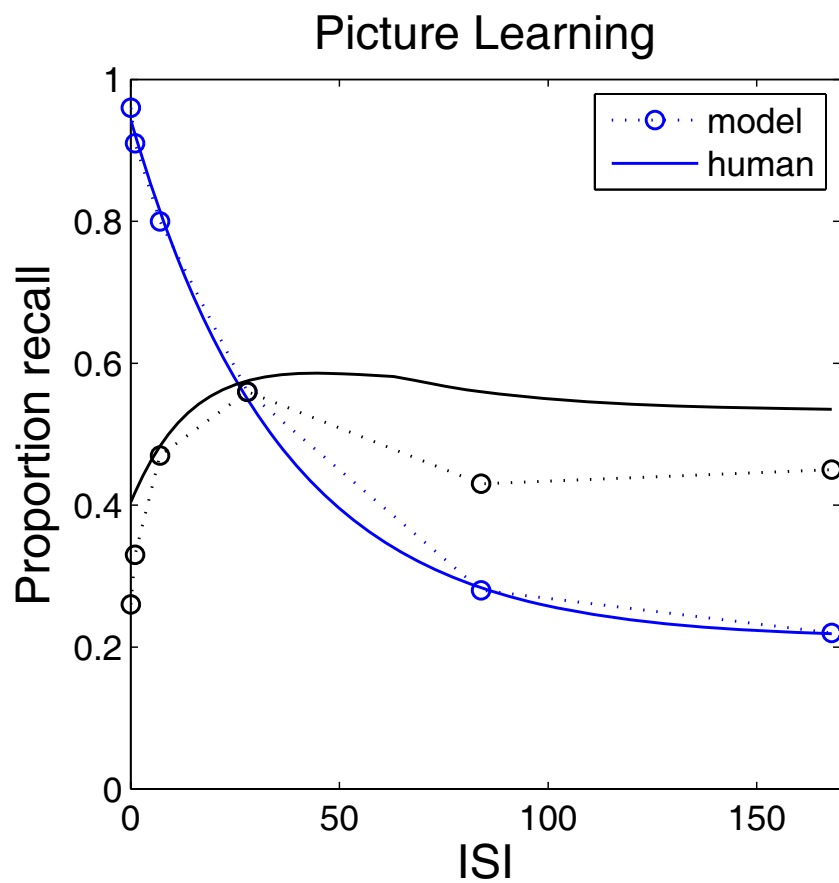
# Cepeda, Coburn, Rohrer, Wixted, Mozer, & Pashler (in press)

**P(recall at study 2)**  
**P(recall at test)**



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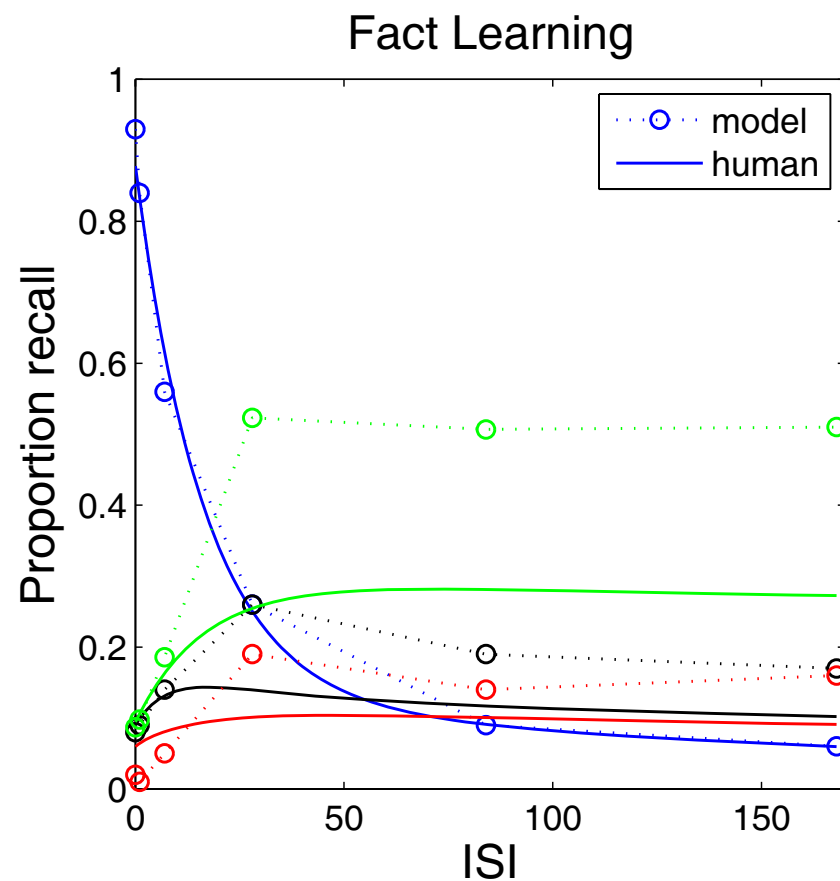
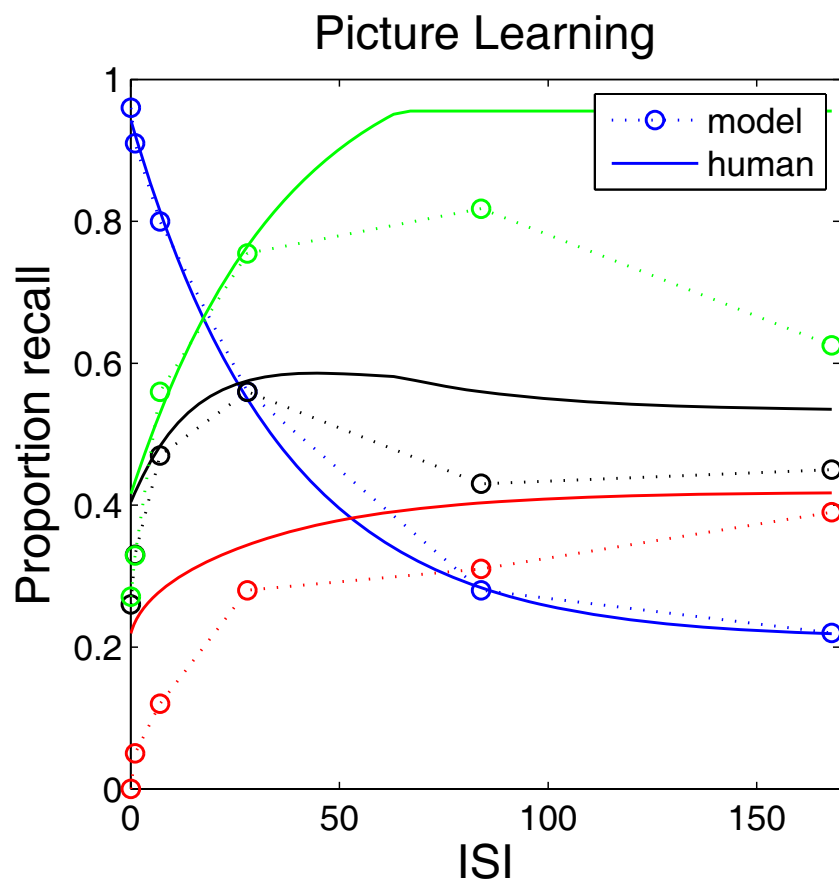
# Cepeda, Coburn, Rohrer, Wixted, Mozer, & Pashler (in press)

$P(\text{recall at study 2})$

$P(\text{recall at test})$

$P(\text{recall at test} \mid \text{recall at study 2})$

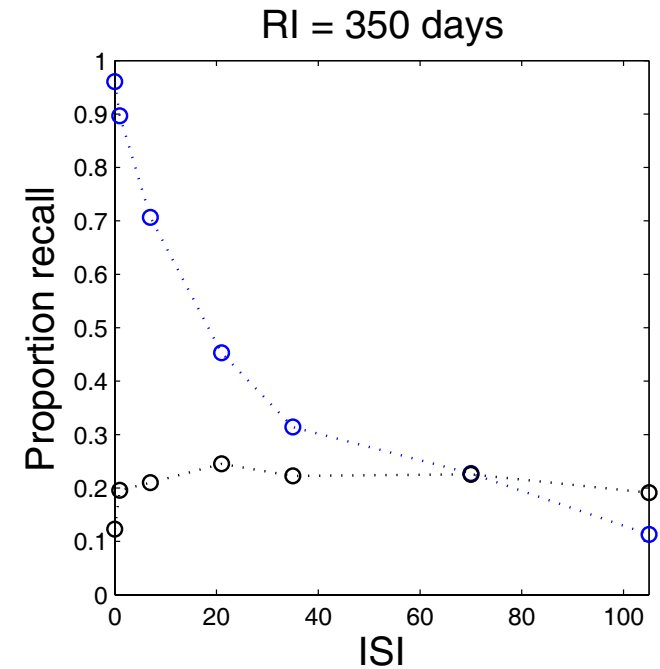
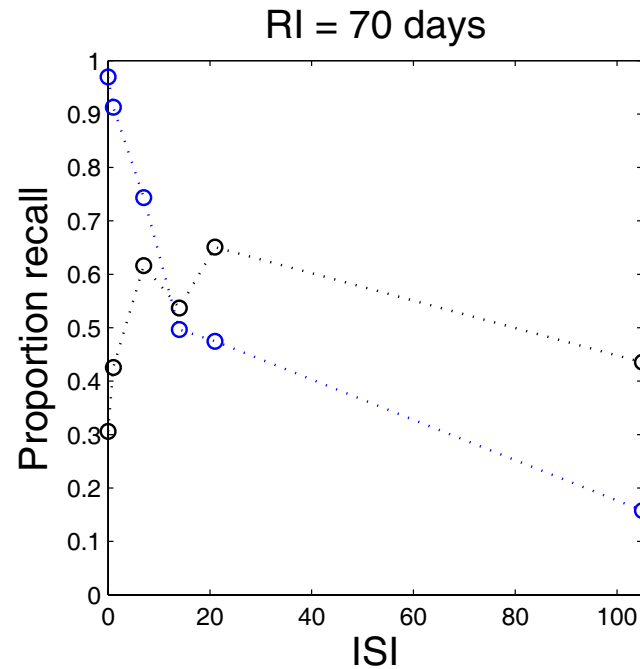
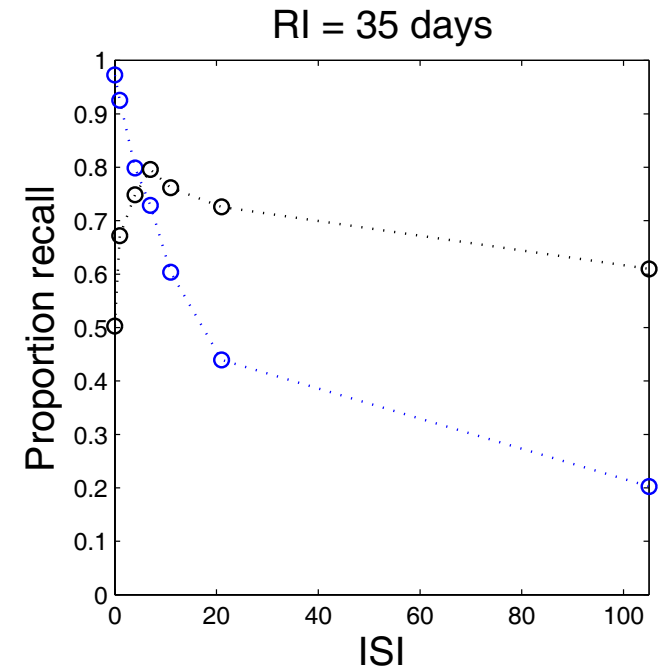
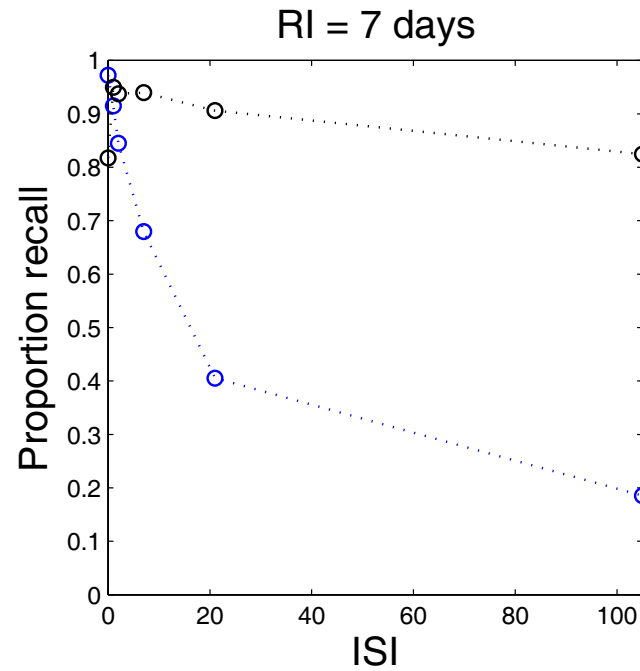
$P(\text{recall at test} \mid \text{no recall at study 2})$



# Simulation of Cepeda, Vul, Rohrer, Wixted, & Pashler (in press)

**P(recall at study 2)**

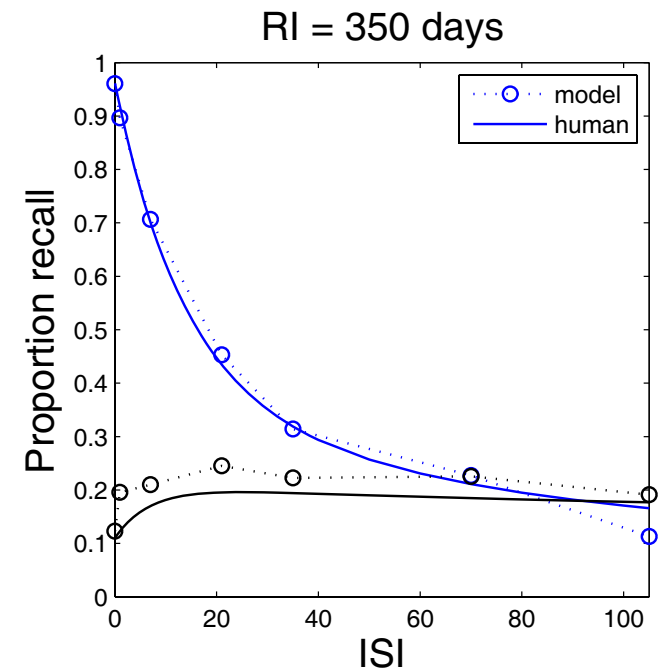
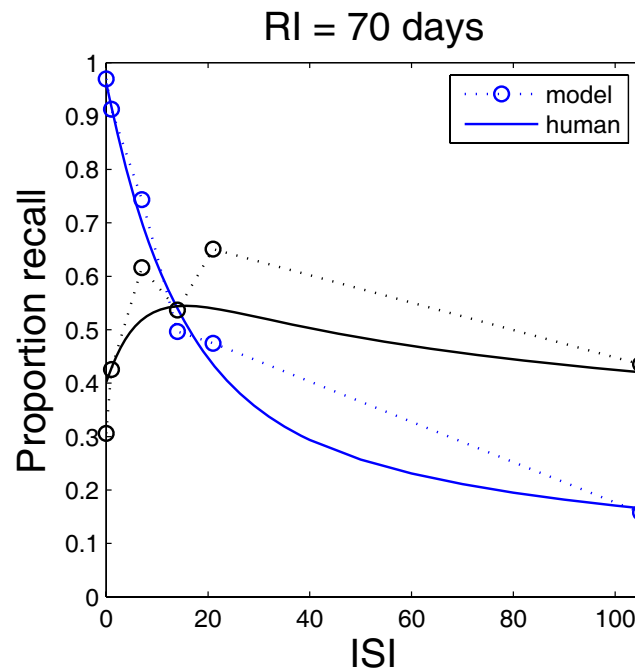
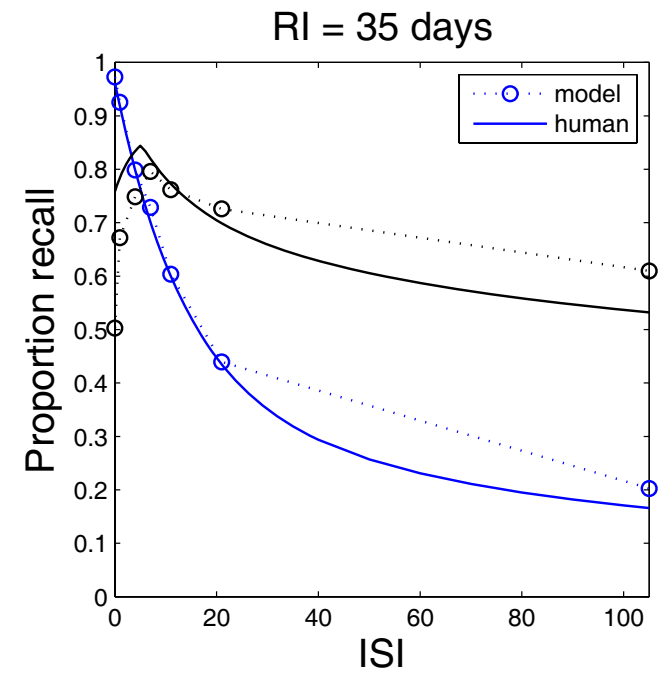
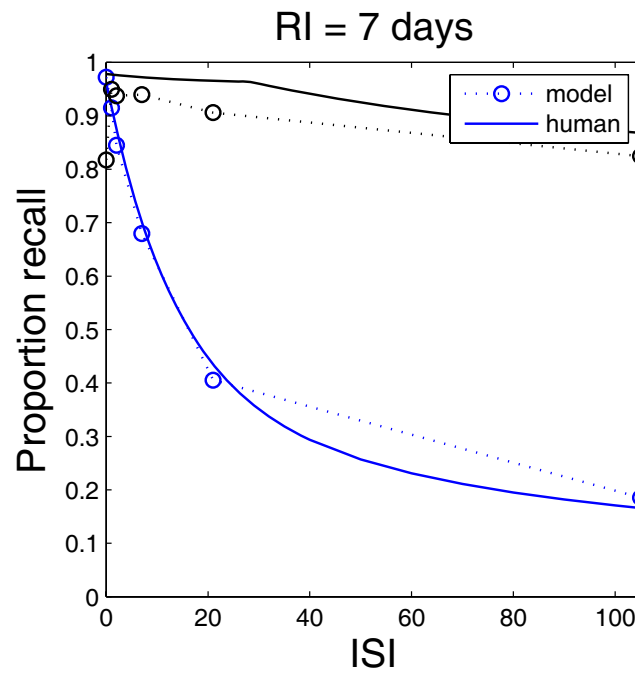
**P(recall at test)**



# Simulation of Cepeda, Vul, Rohrer, Wixted, & Pashler (in press)

**P(recall at study 2)**

**P(recall at test)**



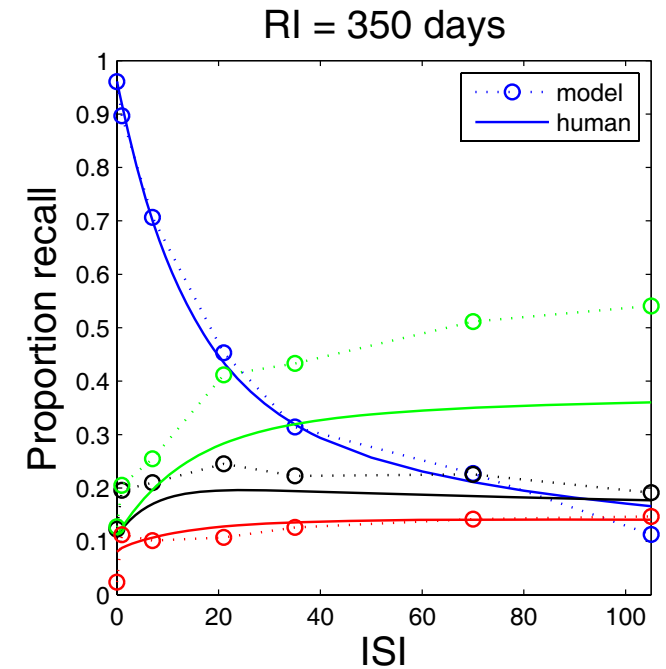
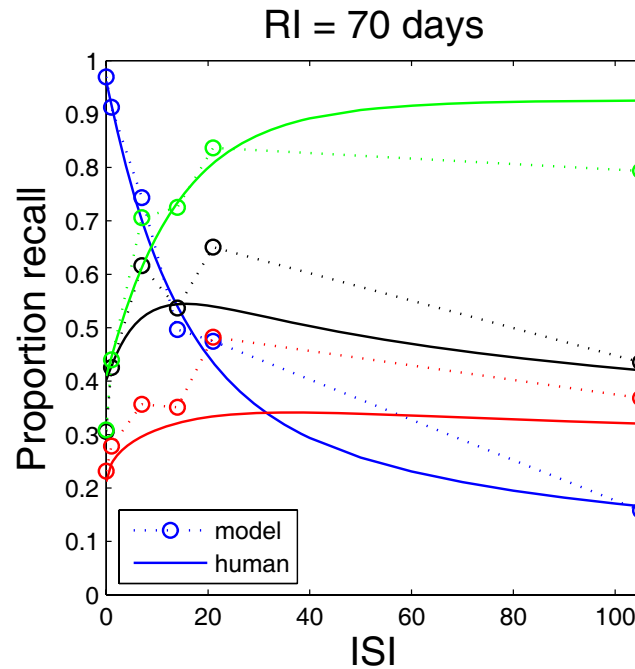
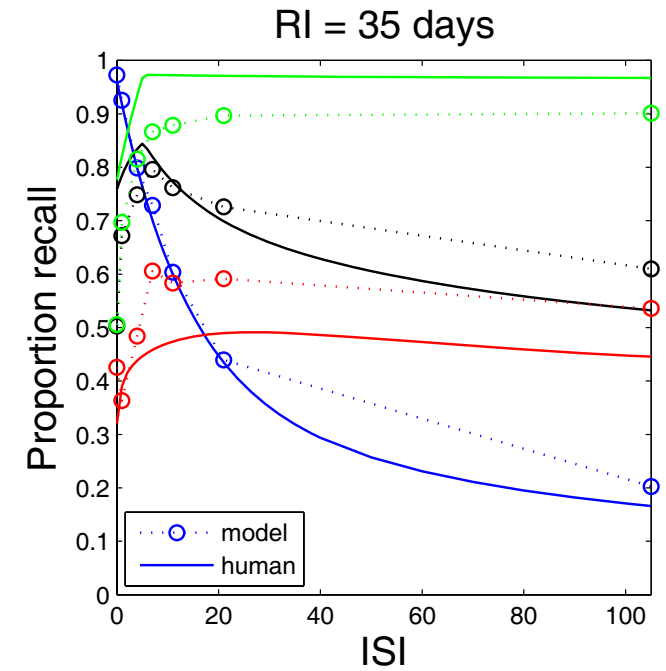
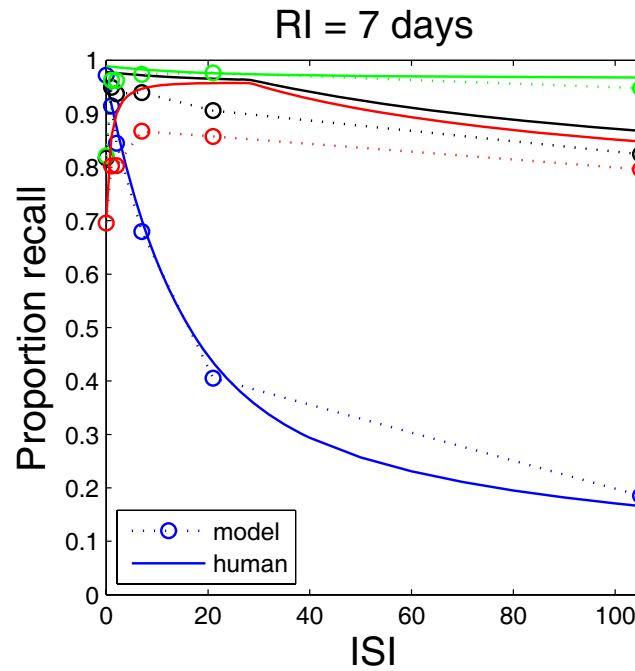
# Simulation of Cepeda, Vul, Rohrer, Wixted, & Pashler (in press)

P(recall at study 2)

P(recall at test)

P(recall at test | recall at study 2)

P(recall at test | no recall at study 2)

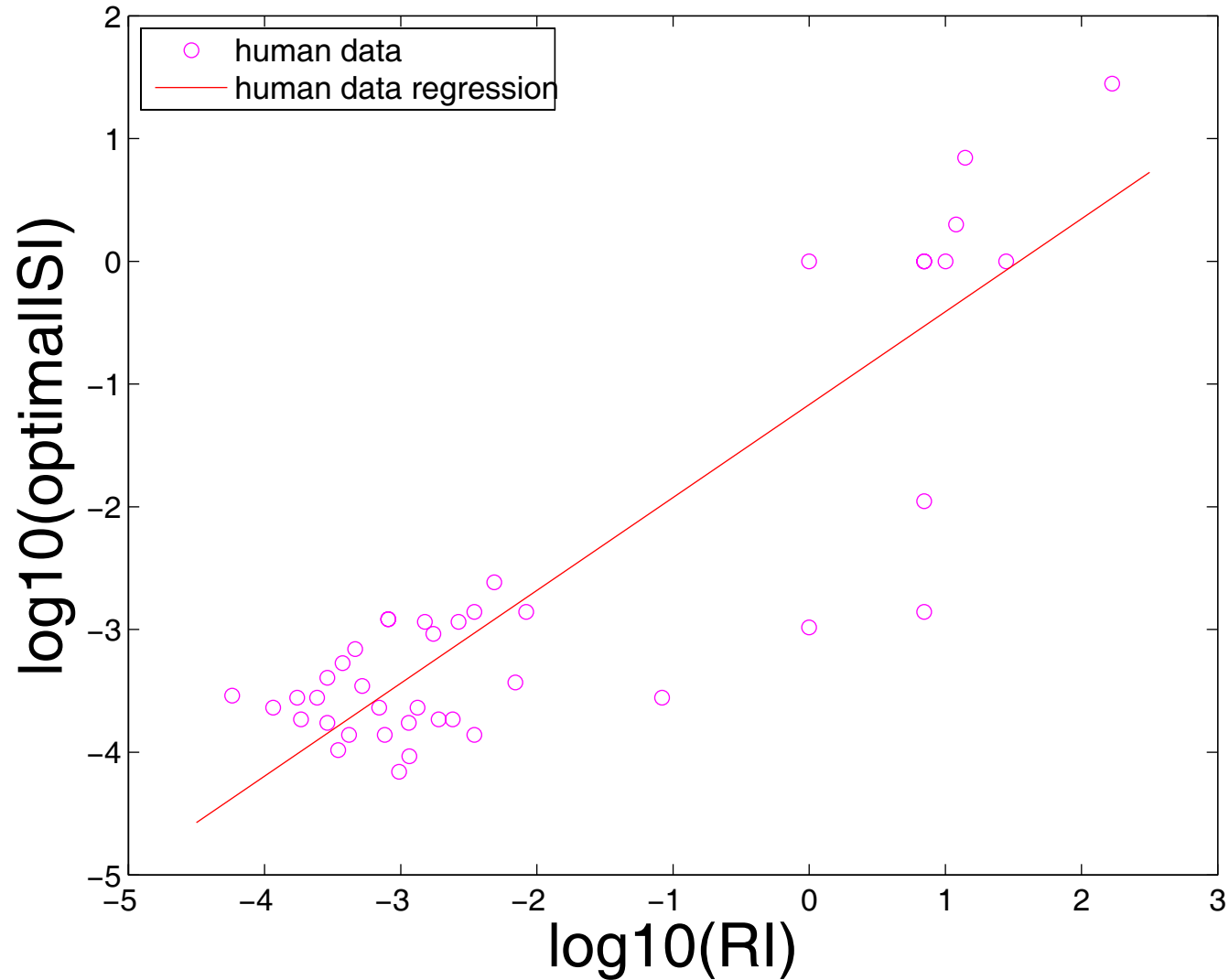


# The Relationship Between RI and Optimal ISI



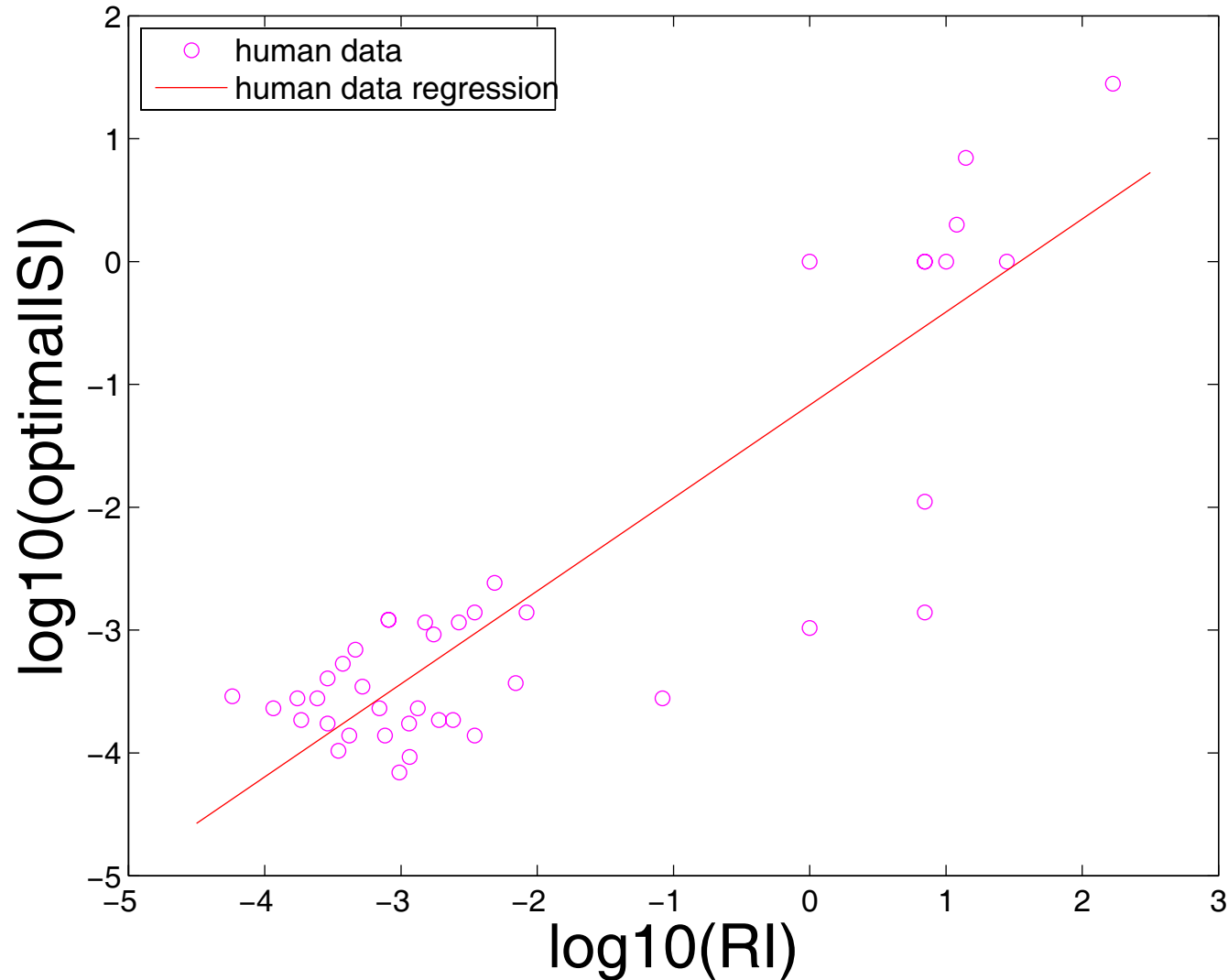
# The Relationship Between RI and Optimal ISI

Cepeda et al. metaanalysis



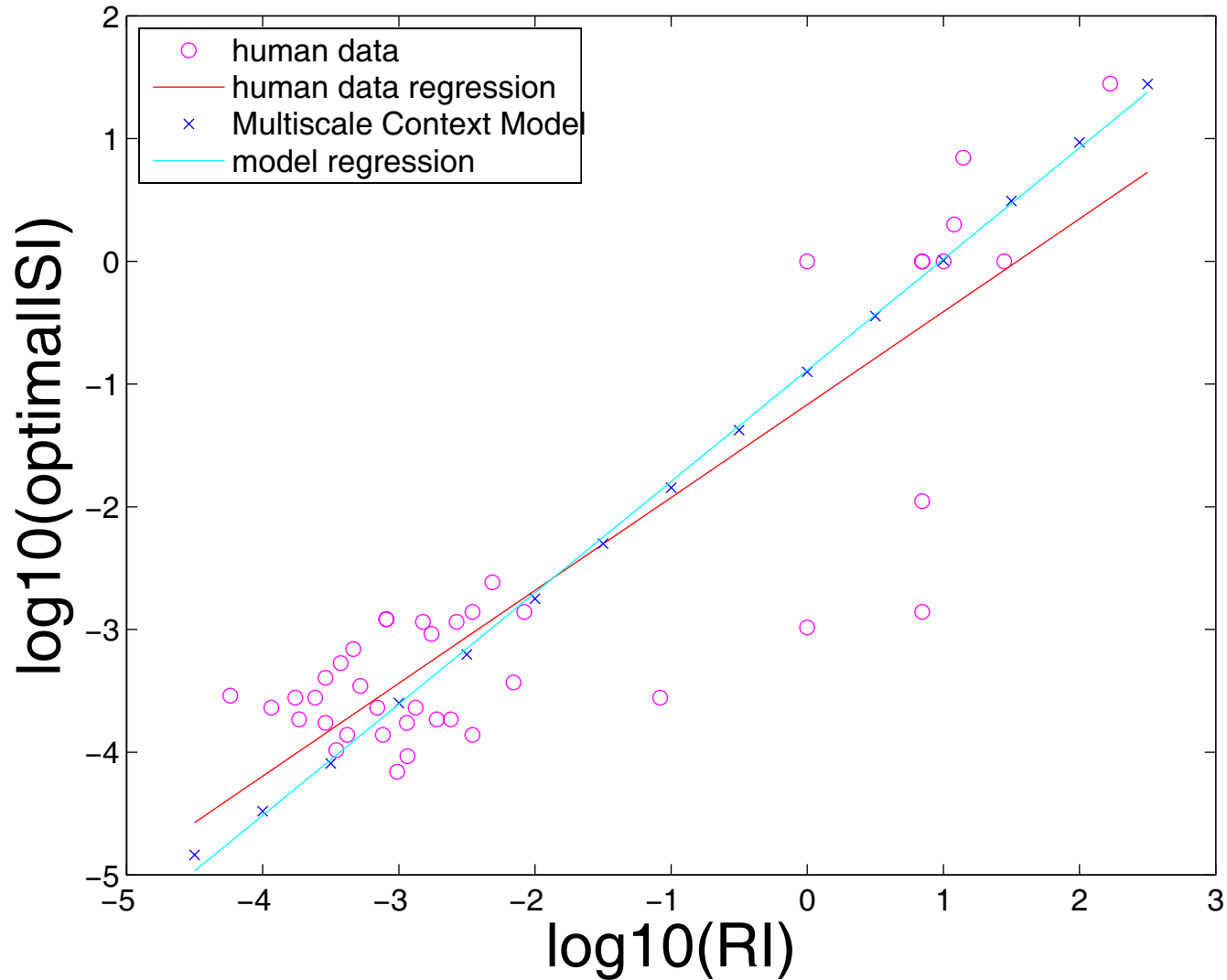
# Simulation of Multiscale Context Model

Random parameter settings of model over a large range



# Simulation of Multiscale Context Model

Random parameter settings of model over a large range



# Why Are We Proposing Yet Another Model?

## Previous models

- have many free parameters, and
- obtain only post hoc fits to data.

## Our goal is to develop a truly predictive model.

Few free parameters

Parameters are fully constrained by the forgetting function

Given forgetting function, optimal distribution of practice can be predicted.

# Current Research

# Current Research

- Exploring DP effects with three study sessions

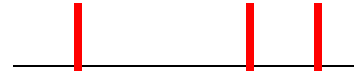
Human study (Kang, Pashler, and Lindsey)

Comparing predictions of two different models  
(Lindsey and Mozer)

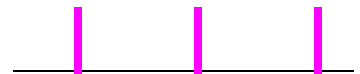
- \* MCM: equal spacing is generally best, but dependent on specific materials

- \* Pavlik & Anderson: decreasing spacing best

decreasing



equal



increasing



# Current Research

- Exploring DP effects with three study sessions

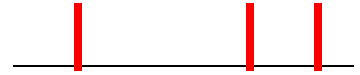
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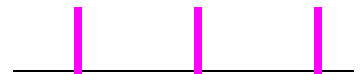
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equal



increasing



- Exploring DP effects with more complex materials

legal, scientific reasoning (Pashler, Coburn, and Carpenter)

# Current Research

- **Exploring DP effects with three study sessions**

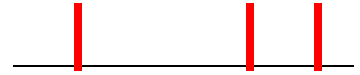
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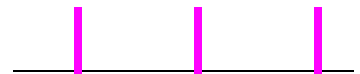
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decreasing



equal



increasing



- **Exploring DP effects with more complex materials**

legal, scientific reasoning (Pashler, Coburn, and Carpenter)

- **Developing Facebook app for learning important facts: survival tactics**

Natural language interface to allow unrestricted answers (Homaei)

Eventually will use MCM to dynamically optimize study session spacing to promote long-term retention (Lindsey and Mozer)



**The End**

